EOSCUBE: A Constraint Database System for High-Level Specification and Efficient Generation of EOSDIS Products

Final Report for Phase 1: Proof-of-Concept

Alexander Brodsky ‡†§
Victor E. Segal ‡†

Consultants:

Jia Chen ‡
Menas Kafatos §
Owen Kelly §
Larry Kerschberg ‡
Samuel Varas ‡
Ruixin Yang §

†CESDIS, Goddard Space Flight Center (GSFC)
NASA

‡Department of Information and Software Engineering (ISE) and
Center for Info. Systems Integration and Evolution (CISIE)
School of Information Technology and Engineering (IT&E)

§Center for Earth Observing and Space Research (CEOSR)
Institute for Computational Sciences and Informatics (CSI)

George Mason University

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Part I
EOSCUBE: Executive Summary

The EOSCUBE constraint database system is designed to be a software productivity tool for high-level specification and efficient generation of EOSDIS and other scientific products. These products are typically derived from large volumes of multidimenstional data which are collected via a range of scientific instruments.

Main Objectives for Phase 1 (Proof-of-concept):

- To demonstrate that EOSCUBE can provide considerable savings in development time of EOSDIS and other scientific products
- To demonstrate that product generation by EOSCUBE from real data sets is feasible.

Ultimate Goals (beyond Phase 1):

Productivity gain: EOSCUBE will allow Earth scientists to compactly specify data products concentrating on their scientific domains, while being relieved from a considerable programming effort.

Interleaved and Optimized Production: EOSCUBE will provide interleaved pipelined evaluation of a series of inter-related products, automatically optimizing dataflow control, buffer management, and materialization supporting clustering and indexing.

Platform Independence: EOSCUBE will support hardware/software platform independence, so that platforms' change would only require changing a small number of interface methods, while leaving products generation software unchanged. It is planned that EOSCUBE will support a mix of underlying object managers, databases, mass storage systems, or just file systems in a very flexible way.

Easy Integration: EOSCUBE is used from within a C++ program and allows to use existing C/C++ code, without the need to translate data types and formats.

Accomplishments in Phase 1:

- Development of the EOSCUBE proof-of-concept prototype based on the CCUBE constraint object-oriented database system
- Specifying in EOSCUBE a range of scientific products, and actually generating a number of them using real input data sets.
- Preparing reports, within this final report, on:
  - Feasibility and productivity study, which contains EOSCUBE specification of a number of scientific products, and test cases run on real data sets
  - Specification of EOSCUBE features and language
  - Architecture and implementation of the EOSCUBE prototype
  - Work in progress on optimizing multi-product generation workflow
  - Recommended course of action

Main Conclusions in Phase 1:

- EOSCUBE has the potential for significant productivity gain in specification and generation of EOSDIS and other scientific products
- Generation of scientific products from real data sets is feasible using the EOSCUBE prototype
- An industrial-strength EOSCUBE implementation will be necessary for deployment and massive use of the system.
- The EOSCUBE language should allow incremental extensions, which are unavoidable in diverse scientific domains
- The overall evaluation model should also support data-flow processing (i.e. pipeline evaluation), in addition to query processing.
- The main aspects of global optimization should deal with interleaved pipelined evaluation of series of inter-related products, and concentrate on optimizing throughput via data flow control, buffer management, and materialization supporting clustering and indexing.

Future Action Paths for EOSCUBE:

We elaborate on recommended activities in Section VI. Below is a summary of main paths of action that will have to be carefully discussed and planned with EOSDIS.

Research Path, including local and global optimization, spatio-temporal indexing and clustering, and GIS constraint algebras

Industrial-strength implementation path, including high-performance EOSCUBE kernel, pipeline evaluation model, ODBC and platforms support, and GIS integration.

Collaborative work with Earth scientists on a specific set of new products, and continued customization of EOSCUBE for them. This will also used as a leverage for later massive deployment of EOSCUBE.

Deployment of EOSCUBE to Centers and Technical Support

Part II
Productivity and Feasibility Study

1 Studied Domains

We have studied the following three domains of processing in order to selects products on which to exemplify EOSCUBE. We next briefly elaborate on each.

Geolocation data processing

This domain includes such procedures as pixel scanning algorithms, radar geometry, and systems of coordinates. The algorithms used in this domain primarily consist of sequences of vector and matrix manipulations, calculations of function values, long summations, and other arithmetic operations. Specifically, we studied the following documents in regards to this domain:

- MODIS Level 1A Earth Location ATBD
- TSDIS algorithm descriptions and equations for combined Geolocation, L1B VIRS and L1B TMI

Conversion of radar measurements into geophysical quantities

This domain directly deals with the radar measurement science. Consequently, data transformations used by the products are mostly large formulas involving mathematical analysis entities such as integration over time and space, the gamma function, trigonometric Al transformations, limits etc. Specifically, we studied the following documents in regards to this domain:

- R.Meneghini and T.Kozu, Spaceborne Weather Radar
- M.Marzoug etc., 'A Class of Single- and Dual- Frequency Algorithms for Rain-Rate Profiling from a Spaceborne Radar. Part I' (TRMM)
- T.Iguchi etc., 'Intercomparison of Single-Frequency Methods for Retrieving a Vertical Rain Profile from Airborne or Spaceborne Radar Data' (TRMM)
- TSDIS Levels 1,2,3 data file specifications
- TSDIS Toolkit User's Guide
- TSDIS Software Design Specifications

Adjustments of geophysical quantities and their aggregations over space and time

This domain involves analysis of streams of measured quantities over time intervals, calculation of more precise values using compositions, adjustments of quantities in regards to other factors present during measuring, standardization of quantities, spatial aggregation and generation of grids of various resolutions. The transformations in this domain generally start from level 3 and go up to the data levels suitable for end-user presentation.

Specifically, we studied the following documents in regards to this domain:

- MODIS Vegetation Index ATBD
- MODIS Land Cover ATBD
- MODIS Infrared Sea Surface Temperature Algorithm
- MODIS Cloud Top Properties and Cloud Phase

We chose primarily this domain to exemplify the use of EOSCUBE. For the reasons of data availability, we decided to use data obtained by the Advanced Very High Resolution Radiometer (AVHRR), which was provided to us by the CEOSR center at GMU. Next we discuss the EOSCUBE databases that was created to store AVHRR data, and which facilitates clustering and secondary storage random access.

2 EOSCUBE Database of AVHRR Data

In the following sections we demonstrate how EOSCUBE can be used to query and produce products from the the following geophysical parameters: *vegetation index*, *cloud amount* and *see surface temperature*. The background for calculation of these parameters has been obtained by studying the documents mentioned above. However, in this section we are not concerned with how the parameters have been computed from the lower-level products. Most queries in the next sections deal with performing computations such as:

- computing parameters for arbitrarily complex areas
- describing spatial and temporal characteristics of areas where parameter(s) satisfy a certain condition
- performing spatial aggregations

- showing area changes over time

A number of products (see details in the next sections) have been actually run on real-data sets (in the AVHRR database) and produced output. The actual data used to run those queries has been collected from several missions using the Advanced Very High Resolution Radiometer (AVHRR). Below are the references to the data sets descriptions:

- Los, S.O., C.O. Justice, C.J. Tucker, 1994. A global 1 by 1 degree NDVI data set for climate studies derived from the GIMMS continental NDVI data. International Journal of Remote Sensing, 15(17):3493-3518.
- Rossow, W.B., L.C. Garder, P.J. Lu and A.W. Walker, 1991. International Satellite Cloud Climatology Project (ISCCP) Documentation of Cloud Data. WMO/TD-No. 266 (revised), World Meteorological Organization, Geneva, 76 pp. plus three appendices. Rossow, W.B., and R.A. Schiffer, 1991: ISCCP cloud data products. Bull. Amer. Meteor. Soc., 72:2-20.
- Reynolds, R. W. and T. M. Smith, 1994. Improved global sea surface temperature analyses. J. Climate, 7:929-948.

The database that we have constructed represent essentially a grid of cells which provide Earth spatial coverage. The grid is a three-dimensional vector with the dimensions representing latitude, longitude, and a time-point (in months), correspondingly. Apart from regular C++ class methods such as constructor and destructor, the grid also contains the following fields and methods:

- `Resolution` is the grid's resolution, 1 degree in our case
- `Low_Lat` is the latitude value corresponding to the (0,0) grid element
- `Low_Lon` is the longitude value corresponding to the (0,0) grid element
- `Lat_Ind_Range(C3_C_RANGE)` gives the range of the latitude index that corresponds to a given rectangle
- `Lon_Ind_Range(C3_C_RANGE)` gives the range of the longitude index that corresponds to a given rectangle
- `Lat_Ind_Upb` is the latitude index upperbound
- `Lon_Ind_Upb` is the longitude index upperbound

Each spatial cell in the grid represents a time series (a vector) of three parametric values:

- NDVI stands for vegetation index, a real number between 0.05 and 0.65. If the value is missing for a particular space-time point, the value of -99 is used instead.
- SST stands for see surface temperature measured in Kelvin. The value of 0 is used for land surfaces.
- CLD stands for cloud amount, a real number between 1 and 100 which has been computed as the average frequency of cloudy pixels. The value of -99 is used for missing data.

There is also a temporal vector of SOI(Southern Oscillation Index) values. Those values do not depend on spatial location and are single values for each time point. Excerpts from the corresponding EOSCUBE declarations appear in Figure 1. The datasets used adhere to the following assumptions:

Spatial Coverage is global. Data in the grid is ordered from North to South and from West to East beginning at 180 degrees West and 90 degrees North. Point (0,0) represents the grid cell centered at 89.5 N and 179.5 W.

Spatial Resolution: The data are given in an equal-angle lat/long grid that has a spatial resolution of 1 X 1 degree lat/long.

Temporal Coverage: January 1987 through December 1988

Temporal Resolution: Monthly mean.

In the products descriptions in the next sections, we will use the following notation:

- `EOSCUBE database of AVHRR data` will denote the input grid, a C++ variable `Grid` of the type `EOS3_Earth_Grid`
- `clin_area` will denote a C++ variable of type `C3_C_LIN`, representing an arbitrary polygon in CST variables *lat*, *lon*
- `dcrange_area` will denote a C++ variable of type `C3_DC_RANGE`, representing a union of rectangles in CST variables *lat*, *lon*
- `crange_area` will denote a C++ variable of type `C3_C_RANGE`, representing a rectangle in CST variables *lat*, *lon*

```
class EOS3_Cell
{
public:
//...
C3_Double NDVI;
C3_Double CLD;
C3_Double SST;
};

class EOS3_Earth__Grid : public C3_Vector<C3_Vector<C3_Vector<EOS3_CELL> > >
{
public:
//...
C3_Int32 Lat_Ind_Upb;
C3_Int32 Lon_Ind_Upb;
C3_Double Resolution;
C3_Double Low_Lat;
C3_Double Low_Lon;
C3_Range_Monoid Lat_Ind_Range( C3_C_RANGE );
C3_Range_Monoid Lon_Ind_Range( C3_C_RANGE );
//...
};

class County
{
public:
//...
C3_String Name;
C3_DC_LIN extent;
//...
};
%
```

Figure 1: EOSCUBE Declarations for AVHRR Grid Schema

- `month` will denote a C++ variable of type `C3_Int32`, which represents a time point, a month index ranging from 0 to 23(for the two-year period).
- `Result` will denote a C++ variable representing the result for each select query. The type of the variable will be different for each product
- MBR stands for Minimum Bounded Rectangle
- `SOI_Time_Series` is a variable of type `C3_Vector<C3_Double>` which represents the SOI temporal vector.

The datasets were provided to us in the form of ASCII files and then imported into EOSCUBE with ObjectStore OODB as underlying storage manager. Below is a sample data fragment representing cloud amount coverage for Jan 1987, extracted from the corresponding file:

```
...
...
58.000  58.000  58.000  58.000  58.000  58.000  58.000  58.000
58.000  58.000  58.000  62.000  66.000  66.000  66.000  66.000
66.000  66.000  66.000  66.000  66.000  66.000  66.000  66.000
66.000  66.000  66.000  66.000  66.000  66.000  66.000  66.000
66.000  66.000  70.000  70.000  70.000  70.000  70.000  70.000
70.000  70.000  70.000  70.000  70.000  70.000  70.000  70.000
70.000  70.000  70.000  70.000  70.000  70.000  70.000  70.000
73.250  76.500  76.500  76.500  76.500  76.500  76.500  76.500
76.500  76.500  76.500  76.500  76.500  76.500  76.500  76.500
76.500  76.500  76.500  76.500  76.500  76.500  76.500  61.500
61.500  61.500  61.500  61.500  61.500  61.500  61.500  61.500
61.500  61.500  61.500  61.500  61.500  61.500  61.500  61.500
61.500  61.500  61.500  61.500  61.500  67.000  72.500  72.500
72.500  72.500  72.500  72.500  72.500  72.500  72.500  72.500
72.500  72.500  72.500  72.500  72.500  72.500  72.500  72.500
72.500  72.500  72.500  72.500  49.000  49.000  49.000  49.000
49.000  49.000  49.000  49.000  49.000  49.000  49.000  49.000
49.000  49.000  49.000  49.000  49.000  49.000  49.000  49.000
49.000  49.000  64.250  79.500  79.500  79.500  79.500  79.500
79.500  79.500  79.500  79.500  79.500  79.500  79.500  79.500
79.500  79.500  79.500  79.500  79.500  79.500  79.500  79.500
...
...
```

3 Products and EOSCUBE Programs

3.1 NDVI Computation over Arbitrary Areas

Calculate NDVI for an arbitrarily input polygon area and a given month.

Input

- `EOSCUBE database of AVHRR data`
- `clin_area` over which NDVI is to be computed
- `month` for which NDVI is to be computed

Output

`Result`, of type C3_Double, a real value representing the vegetation index for `area` at time point `tp`, calculated using the mean average

Description

An MBR for `clin_area` is computed, then iteration over the grid's window corresponding to the MBR is performed using the `Lat_Ind_Range` and `Lon_Ind_Range` methods. For each cell the TV predicate checks if the cell is actually inside the area. Each C-RANGE is then checked on intersection with `clin_area`. The NDVI for the time point `month` is inserted into result, which is of the special averaging primitive monoid type.

Sample Input

```
C3_C_LIN clin_area =   ( 3 * lon - lat >= 51 )
                    && ( lat - lon <= 70 )
                    && ( lon <= 65 )
                    && ( lat >= 110 );
C3_Int32 month = 0;
```

Output For The Sample Input

See Appendix A.

Code

```
EXEC COMPREH
SELECT { cell.NDVI } INTO { C3_Avg<double>} Result
   DEFINE area_mbr AS { clin_area.Get_MBR() }
FROM {Grid.Lat_Ind_Range(area_mbr )} AS {C3_Int32} lat_ind
FROM {Grid.Lon_Ind_Range(area_mbr )} AS {C3_Int32} lon_ind
   DEFINE delta AS { (Grid.Resolution)/2.0 }
   DEFINE center_lat AS { Grid.Low_Lat+(Grid.Resolution)*lat_ind + delta}
   DEFINE center_lon AS { Grid.Low_Lon+(Grid.Resolution)*lon_ind + delta}
WHERE { clin_area.TV( (lat/(center_lat), lon/(center_lon) ) ) }
   DEFINE cell AS { Grid[lat_ind][lon_ind][month] }
WHERE { cell.NDVI >= (C3_Double)0 }
;
END COMPREH
```

3.2 Computing Areas with SST and Clouds Satisfying Conditions

Compute the spatial specifications of SST and CA data which satisfy the condition $290K \leq SST \leq 300K$ AND $CA \geq 50$

Input

- `EOSCUBE database of AVHRR data`
- `cld_lwb`, `sst_lwb`, and `sst_upb`, of type `C3_Double`
- `month`

Output

`Result`, of type `C3_DC_RANGE` an area at time point `month`, with $sst\_lwb \leq SST \leq sst\_upb$ AND $CA \geq cld\_lwb$

Description

Iteration over the whole grid is performed. If the condition on SST and CLD is satisfied for a cell, the C_RANGE corresponding to the cell is constructed and added to the `Result`; the latter is of type `C3_DC_RANGE` which represents a union of rectangles.

Sample Input

```
C3_Int32 month = 0;
C3_Double cld_lwb = 70.0;
C3_Double sst_lwb = 200.0;
C3_Double sst_upb = 300.0;
```

Output For The Sample Input

See the output image in Appendix A.

Code

```
EXEC COMPREH
SELECT { square } INTO { C3_DC_RANGE } Result
FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lat_Ind_Upb,1)} AS {C3_Int32} lat_in
FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lon_Ind_Upb,1)} AS {C3_Int32} lon_in
WHERE { Grid[lat_ind][lon_ind][month].CLD >= cld_lwb }
WHERE { Grid[lat_ind][lon_ind][month].SST >= sst_lwb }
WHERE { Grid[lat_ind][lon_ind][month].SST <= sst_upb }
   DEFINE delta AS { (Grid.Resolution)/2.0 }
   DEFINE center_lat AS { Grid.Low_Lat+(Grid.Resolution)*lat_ind + delta}
   DEFINE center_lon AS { Grid.Low_Lon+(Grid.Resolution)*lon_ind + delta}
   DEFINE square AS { (center_lat - delta <= lat <=  center_lat + delta)
                      && (center_lon - delta <= lon <=  center_lon + delta) }
;
END COMPREH
```

3.3 Computing Area with SST Changes over Time

For two given months, find the spatial area where SST has increased by `sst_delta`.

Input

- `EOSCUBE database of AVHRR data`
- `month_1`
- `month_2`
- `sst_delta`, of type `C3_Double`

Output

`Result`, of type `C3_DC_RANGE` , an area where the SST has increased at least by `sst_delta` from time point `month_1` to time point `month_2`

Description

Iteration over the whole grid is performed. If the condition on SST for the two given months is satisfied for a cell, the C_RANGE corresponding to the cell is constructed and added to the `Result`; the latter is of type `C3_DC_RANGE` which represents a union of rectangles.

Sample Input

```
C3_Int32 month_1 = 0;
C3_Int32 month_2 = 1;
C3_Double sst_delta = 1.0;
```

Output For The Sample Input

See Appendix A.

Code

```
EXEC COMPREH
SELECT { square } INTO { C3_DC_RANGE } Result
FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lat_Ind_Upb,1)} AS {C3_Int32} lat_in
FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lon_Ind_Upb,1)} AS {C3_Int32} lon_in
WHERE { Grid[lat_ind][lon_ind][month_2].SST -
                              Grid[lat_ind][lon_ind][month_1].SST >= sst_delta }
   DEFINE delta AS { (Grid.Resolution)/2.0 }
   DEFINE center_lat AS { Grid.Low_Lat+(Grid.Resolution)*lat_ind + delta}
   DEFINE center_lon AS { Grid.Low_Lon+(Grid.Resolution)*lon_ind + delta}
   DEFINE square AS { (center_lat - delta <= lat <=  center_lat + delta)
                        && (center_lon - delta <= lon <=  center_lon + delta) }
;
END COMPREH
```

3.4 Time Series of Spatial Correlation between Shifted Areas - Version 1

For a selected a spatial region and a latitude/longitude shifts, (with the shifts, another region is defined; the two regions have the same shape and same size), compute a time series, that, for each month, will hold the spatial correlation (point to point) between SST over region 1 and NDVI over region 2.

Input

- `EOSCUBE database of AVHRR data`
- `clin_area`
- `lat_shift`, of type `C3_Double`
- `lon_shift`, of type `C3_Double`

Output

`Result`, of type `C3_Vector<C3_Double>` , a vector of real values representing spatial correlation for each month.

Description

The main query iterates over time first and uses a custom spatial correlation function, `area_corr`, which is implemented here

Sample Input

`N/A`

Output For The Sample Input

N/A

Code

```
EXEC COMPREH
SELECT { area_corr( area, month) }
INTO { C3_Vector<C3_Double>} Result
FROM { C3F_Range_Mon(0, Max_Month-1)} AS {C3_Int32} month
;END COMPREH
```

This computation uses the function area_corr, which is defined next:

```
C3_Double area_corr ( C3_C_LIN area, C3_Int32 month ){
sprod = sum_ndvi = sum_sst = sum_ndvi_sq = sum_sst_sq = 0;

EXEC COMPREH
PERFORM { sprod += cell.NDVI * new_cell.SST;
          sum_ndvi  += cell.NDVI;
          sum_sst += cell.SST;
          sum_ndvi_sq += cell.NDVI* cell.NDVI;
          sum_sst_sq += cell.SST* cell.SST;
        }
FROM { C3F_Range_Mon(0, Grid.Lat_Ind_Upb)} AS lat_ind
FROM { C3F_Range_Mon(0, Grid.Lon_Ind_Upb)} AS lon_ind
     DEFINE lat_val AS { Grid.Low_Lat+(Grid.Resolution)*lat_ind}
     DEFINE lon_val AS { Grid.Low_Lon+(Grid.Resolution)*lon_ind}
WHERE {area.TV(lat/lat_val, lon/lon_val)}
     DEFINE new_lat_ind AS {lat_ind + lat_shift/Grid.Resolution}
     DEFINE new_lon_ind AS {lon_ind + lon_shift/Grid.Resolution}
     DEFINE cell AS { Grid[lat_ind][lon_ind][month] }
     DEFINE new_cell AS { Grid[new_lat_ind][new_lon_ind][month] }
END COMPREH

C3_Double n = (Grid.Lat_Ind_Upb+1) * (Grid.Lon_Ind_Upb+1)
C3_Double ndvi_avg = sum_ndvi/n;
C3_Double sst_avg = sum_sst/n;
double corr = ( sprod/n - ndvi_avg*sst_avg )/ (sqrt
     ( sum_ndvi_sq/n-ndvi_avg*ndvi_avg)*sqrt
     ( sum_sst_sq/n-sst_avg*sst_avg))
return (corr)
}
```

3.5 Time Series of Spatial Correlation between Shifted Areas - Version 2

This is another (number 2) implementation of the spatial correlation product

Input

- `EOSCUBE database of AVHRR data`
- `clin_area`
- `lat_shift`, of type `C3_Double`
- `lon_shift`, of type `C3_Double`

Output

`Result`, of type `C3_Vector<C3_Double>` , a vector of real values representing spatial correlation for each month.

Description

Iteration over time goes first and materializes correlation sequences for each iteration. A generic correlation function, `correl` is implemented here.

Sample Input

`N/A`

Output For The Sample Input

N/A

Code

```
EXEC COMPREH
SELECT { correl(corr_seqs) }
INTO {C3_Vector<C3_Double> Result
FROM { C3F_Range_Mon(0, Max_Month-1)} AS {C3_Int32} month
DEFINE corr_seqs AS { area_corr_seqs(area,month) }
;
END COMPREH



C3_Vector<C3_Pair<C3_Double,C3_Double>>
area_corr_seqs( C3_C_LIN area, C3_Int32 month )
{
EXEC COMPREH
SELECT { C3_Pair<C3_Double,C3_Double>(cell.NDVI, new_cell.SST)
INTO {C3_Vector<C3_Pair<C3_Double,C3_Double>>} Result
FROM { C3F_Range_Mon(0, Grid.Lat_Ind_Upb)} AS lat_ind
FROM { C3F_Range_Mon(0, Grid.Lon_Ind_Upb)} AS lon_ind
     DEFINE lat_val AS { Grid.Low_Lat+(Grid.Resolution)*lat_ind}
     DEFINE lon_val AS { Grid.Low_Lon+(Grid.Resolution)*lon_ind}
WHERE {area.TV(lat/lat_val, lon/lon_val)}
     DEFINE new_lat_ind AS {lat_ind + lat_shift/Grid.Resolution}
     DEFINE new_lon_ind AS {lon_ind + lon_shift/Grid.Resolution}
     DEFINE cell AS { Grid[lat_ind][lon_ind][month] }
     DEFINE new_cell AS { Grid[new_lat_ind][new_lon_ind][month] }
;
END COMPREH
return Result;
};

C3_Double correl( C3_Vector<C3_Pair<C3_Double,C3_Double>> corr_seqs )
{
xysum = xsum = ysum = x2sum = y2sum = 0;
PERFORM {xysum += x*y;
         xsum  += x;
         ysum += y;
         x2sum += x*x;
         y2sum += y*y;
```

```
        }
FROM { corr_seqs} AS pair
     DEFINE x AS pair.First
     DEFINE y AS pair.Second
END COMPREH

C3_Double n = corr_seqs.Count();
C3_Double xavg = xsum/n;
C3_Double yavg = ysum/n;
double corr = (xysum/n - xavg*yavg) /
                (sqrt(x2sum/n-xavg*xavg)*sqrt(y2sum/n-yavg*yavg))
return ( corr )
};
```

3.6 Time Series of Spatial Correlation between Shifted Areas - Version 3

Input

- `EOSCUBE database of AVHRR data`
- `clin_area`
- `lat_shift`, of type `C3_Double`
- `lon_shift`, of type `C3_Double`

Output

`Result`, of type `C3_Vector<C3_Double>` , a vector of real values representing spatial correlation for each month.

Description

Same as the previous product, but correlation sequences are not materialized; rather special monoid is used which incrementally implements correlation.

Sample Input

`N/A`

Output For The Sample Input

N/A

Code

```
EXEC COMPREH
SELECT { area_corr(area, month) }
INTO {C3_Vector<C3_Double> Result
FROM { C3F_Range_Mon(0, Max_Month-1)} AS {C3_Int32} month
;
END COMPREH


C3_Double area_corr ( C3_C_LIN area, C3_Int32 month )
{
EXEC COMPREH
SELECT { C3_Pair<C3_Double,C3_Double>(cell.NDVI, new_cell.SST)
INTO {C3_Correl<C3_Double>} Result
FROM { C3F_Range_Mon(0, Grid.Lat_Ind_Upb)} AS lat_ind
FROM { C3F_Range_Mon(0, Grid.Lon_Ind_Upb)} AS lon_ind
     DEFINE lat_val AS { Grid.Low_Lat+(Grid.Resolution)*lat_ind}
     DEFINE lon_val AS { Grid.Low_Lon+(Grid.Resolution)*lon_ind}
WHERE {area.TV(lat/lat_val, lon/lon_val)}
     DEFINE new_lat_ind AS {lat_ind + lat_shift/Grid.Resolution}
     DEFINE new_lon_ind AS {lon_ind + lon_shift/Grid.Resolution}
     DEFINE cell AS { Grid[lat_ind][lon_ind][month]. }
     DEFINE new_cell AS { Grid[new_lat_ind][new_lon_ind][month] }
;
END COMPREH
return Result;
}
```

3.7 Time Series of Spatial Correlation between Shifted Areas - Version 4

Input

- EOSCUBE database of AVHRR data
- clin_area
- lat_shift, of type C3_Double
- lon_shift, of type C3_Double

Output

Result, of type C3_Vector<C3_Double> , a vector of real values representing spatial correlation for each month.

Description

This more efficient version iterate over cells first and computes all correlation values simultaneously

Sample Input

```
C3_C_LIN clin_area = ( 100 <= lat <= 110  &&  50 <= lon <= 60 ); // part of No
C3_Double lon_shift = -60 ; // into Pacific ocean
C3_Double lat_shift = -20 ;
```

Output For The Sample Input

See Appendix A.

Code

```
C3_Correl<double> Result[Max_Months];

EXEC COMPREH
PERFORM
   DEFINE area_mbr AS { clin_area.Get_MBR() }
FROM {Grid.Lat_Ind_Range(area_mbr )} AS {C3_Int32} lat_ind
PERFORM { C3M_TRACE(lat_ind) }
FROM {Grid.Lon_Ind_Range(area_mbr )} AS {C3_Int32} lon_ind
   DEFINE delta AS { (Grid.Resolution)/2.0 }
   DEFINE center_lat AS { Grid.Low_Lat+(Grid.Resolution)*lat_ind + delta}
   DEFINE center_lon AS { Grid.Low_Lon+(Grid.Resolution)*lon_ind + delta}
WHERE { clin_area.TV( (lat/(center_lat), lon/(center_lon) ) ) }
   DEFINE new_lat_ind AS {lat_ind + lat_shift/Grid.Resolution}
   DEFINE new_lon_ind AS {lon_ind + lon_shift/Grid.Resolution}
   DEFINE cell_ts AS { Grid[lat_ind][lon_ind] }
   DEFINE new_cell_ts AS { Grid[new_lat_ind][new_lon_ind] }
  SELECT {C3_Pair<double,double>(cell_ts[month].NDVI,
                                 new_cell_ts[month].SST)}
  INTO correl
  FROM {C3F_Range_Monoid<C3_Int32>(0,Max_Months-1,1)} AS {C3_Int32} month
     DEFINE correl AS { Result[month] }
  ;
;
END COMPREH
```

3.8 Temporal Correlation

For time periods when SOI < 0 (or $>= 0$), compute the spatial region over which the temporal correlation between NDVI and SOI is equal to or greater than a given value `tc_lwb`.

Input

- `EOSCUBE database of AVHRR data`
- `tc_lwb`, of type `C3_Double`

Output

`Result`, of type `C3_DC_RANGE`, an area where the temporal correlation between SOI and NDVI over the 24 months is at least `tc_lwb`

Description

straightforward

Sample Input

```
C3_Double tc_lwb = 0.6 ;
```

Output For The Sample Input

See the outpour image in Appendix A.

Code

```
EXEC COMPREH
SELECT { square } INTO { C3_DC_RANGE } Result
FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lat_Ind_Upb,1)} AS {C3_Int32} lat_i
FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lon_Ind_Upb,1)} AS {C3_Int32} lon_in
   DEFINE cell_ts AS { Grid[lat_ind][lon_ind] }
  SELECT { C3_Pair<double,double>( cell_ts[month].NDVI,
                                   SOI_Time_Series[month] ) }
  INTO {C3_Correl<double>} correl
  FROM { C3F_Range_Monoid<C3_Int32>(0, Max_Months-1,1)} AS {C3_Int32} month
  WHERE { cell_ts[month].NDVI >= 0.0 }
  WHERE { SOI_Time_Series[month] >= 0.0 }
  ;
WHERE {  (double)correl >= tc_lwb }
   DEFINE delta AS { (Grid.Resolution)/2.0 }
   DEFINE center_lat AS { Grid.Low_Lat+(Grid.Resolution)*lat_ind + delta}
   DEFINE center_lon AS { Grid.Low_Lon+(Grid.Resolution)*lon_ind + delta}
   DEFINE square AS { (center_lat - delta <= lat <=  center_lat + delta)
                  && (center_lon - delta <= lon <=  center_lon + delta) }
;
END COMPREH
```

3.9 Spatial Aggregation

Produce a new grid which is $N * N$ times more coarse for a given `month`.

Input

- `EOSCUBE database of AVHRR data`
- `N`, of type `C3_Int32`
- `month`

Output

`Result`, of type `EOS3_Earth_Grid` , a new coarse grid with the resolution `N` the input grid resolution(at time point `month`), and NDVI values averaged for each coarse cell over finer cells inside it.

Description

Iteration over coarse cells goes first, then corresponding finer cells are visited and NDVI is averaged using the averaging monoid

Sample Input

```
C3_Int32 month = 0;
C3_Int32 N = 10;
```

Output For The Sample Input

See Appendix A.

Code

```
EOS3_Earth_Grid New_Grid( 360/N, 180/N );
New_Grid.Size_All();

EXEC COMPREH
PERFORM
FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lat_Ind_Upb/N, 1)}
       AS {C3_Int32} new_lat_ind
FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lon_Ind_Upb/N, 1)}
                                          AS {C3_Int32} new_lon_ind
   SELECT { cell.NDVI }
   INTO {C3_Avg<double>} avg_ndvi
   FROM { C3F_Range_Monoid<C3_Int32>(new_lat_ind*N, (new_lat_ind+1)*N-1, 1)}
                                          AS {C3_Int32} lat_ind
   FROM { C3F_Range_Monoid<C3_Int32>(new_lon_ind*N, (new_lon_ind+1)*N-1, 1)}
                                          AS {C3_Int32} lon_ind
      DEFINE cell AS {Grid[lat_ind][lon_ind][month]}
   WHERE { cell.NDVI >= 0.0 }
   ;
PERFORM {New_Grid[new_lat_ind][new_lon_ind][month].NDVI = (C3_Double)avg_ndvi
;
END COMPREH
```

3.10 NDVI Animation Movie

Produce a sequence of areas with NDVI at least `ndvi_lwb` for 24 months.

Input

- `EOSCUBE database of AVHRR data`
- `ndvi_lwb`, of type `C3_Double`

Output

`Result`, of type `C3_Vector<C3_DC_LIN>`, representing a collection of areas with NDVI equal or greater than `ndvi_lwb`

Description

Iteration over all time points is performed, then `NDVI_LESS(ndvi_lwp)` is called on each iteration

Sample Input

```
C3_Double ndvi_lwb = 0.4;
```

Output For The Sample Input

See Appendix A + animation in the demo.

Code

%

```
EXEC COMPREH
SELECT { frame }
INTO {C3_Vector<C3_DC_RANGE>} Result
FROM {C3F_Range_Monoid<C3_Int32>(0,Max_Months-1,1)} AS {C3_Int32} month
   SELECT { square } INTO { C3_DC_RANGE } frame
   FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lat_Ind_Upb, 1)}
                                                  AS {C3_Int32} lat_ind
   FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lon_Ind_Upb, 1)}
                                                  AS {C3_Int32} lon_ind
   WHERE { Grid[lat_ind][lon_ind][month].NDVI >= ndvi_lwb }
      DEFINE delta AS { (Grid.Resolution)/2.0 }
      DEFINE center_lat AS { Grid.Low_Lat+(Grid.Resolution)*lat_ind + delta}
      DEFINE center_lon AS { Grid.Low_Lon+(Grid.Resolution)*lon_ind + delta}
      DEFINE square AS { (center_lat - delta <= lat <=  center_lat + delta)
                         && (center_lon - delta <= lon <=  center_lon + delta
   ;
;
END COMPREH
```

3.11 Temporal Correlation and SST Anomaly

Compute the spatial area in which the temporal correlation between SST Anomaly and CA is greater than or equal to `tc_lwb`.

Input

- `EOSCUBE database of AVHRR data`
- `tc_lwb`

Output

`Result` of type `C3_DC_RANGE` which represents the computed area.

Description

Note, anomaly here is a difference between a SST value and SST 2-year average.

Sample Input

N/A

Output For The Sample Input

N/A.

Code

```
EXEC COMPREH
SELECT { square } INTO { C3_DC_RANGE } Result
FROM { C3F_Range_Mon<Int32>(0, Grid.Lat_Ind_Upb,1)} AS {Int32} lat_ind
FROM { C3F_Range_Mon<Int32>(0, Grid.Lon_Ind_Upb,1)} AS {Int32} lon_ind
      SELECT { Grid[lat_ind][lon_ind][month].SST }
      INTO {C3_Avg<double>} sst_avg
      FROM { C3F_Range_Mon<Int32}=>(0, Max_Month-1,1)}  AS {Int32} month
      WHERE { cell_ts[month].SST >= 0 }
      ;
      SELECT { C3_Pair<double,double>( cell_ts[month].CLD, sst_anomaly ) }
      INTO {C3_Correl<double>} correl
      FROM { C3F_Range_Mon<Int32>(0, Max_Month-1,1)} AS {Int32} month
      WHERE { cell_ts[month].CLD >= 0 }
      WHERE { cell_ts[month].SST >= 0 }
          DEFINE sst_anomaly AS {  Grid[lat_ind][lon_ind][month].SST-sst_avg}
      ;
WHERE {  (double)correl >= correl_lwb }
      DEFINE delta AS { (Grid.Resolution)/2.0 }
      DEFINE center_lat AS { Grid.Low_Lat+(Grid.Resolution)*lat_ind + delta}
      DEFINE center_lon AS { Grid.Low_Lon+(Grid.Resolution)*lon_ind + delta}
      DEFINE square AS { (center_lat - delta <= lat <=  center_lat + delta)
                && (center_lon - delta <= lon <=  center_lon + delta)}
;
END COMPREH
```

3.12 Color World Map

Create three spatial areas: green, yellow and red, classified by NDVI ranges, per given `month`

Input

- `EOSCUBE database of AVHRR data`
- `month`

Output

- `green_area` of type `C3_DC_RANGE`
- `yellow_area` of type `C3_DC_RANGE`
- `red_area` of type `C3_DC_RANGE`

Description

Straightforward.

Sample Input

N/A

Output For The Sample Input

N/A

Code

```
C3_DC_RANGE green_area, yellow_area, red_area

EXEC COMPREH
SELECT square(lat_ind, lon_ind, Grid.res)
INTO {C3_DC_RANGE} green_area
FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lat_Ind_Upb,1)} AS {C3_Int32} lat_in
FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lon_Ind_Upb,1)} AS {C3_Int32} lon_i
WHERE {Grid[lat_ind][lon_ind][month].NDVI <= green_NDVI_bound}
;

SELECT square(lat_ind, lon_ind, Grid.res)
INTO {C3_DC_RANGE} yellow_area
FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lat_Ind_Upb,1)} AS {C3_Int32} lat_i
FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lon_Ind_Upb,1)} AS {C3_Int32} lon_in
WHERE {Grid[lat_ind][lon_ind][month].NDVI > green_NDVI_bound &&
      Grid[lat_ind][lon_ind][month].NDVI <= yellow_NDVI_bound}

;
SELECT square(lat_ind, lon_ind, Grid.res)
INTO {C3_DC_RANGE} red_area
FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lat_Ind_Upb,1)} AS {C3_Int32} lat_i
FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lon_Ind_Upb,1)} AS {C3_Int32} lon_in
WHERE {Grid[lat_ind][lon_ind][month].NDVI > yellow_NDVI_bound}
;
END COMPREH

C3_C_RANGE square(int lat_ind, int lon_ind, double res);
{C3_CST_Var lat, lon;
return(Grid.lat_low + lat_ind*res <= lat &&
                Grid.lat_low + (lat_ind + 1)*res &&
                Grid.lon_low + lon_ind*res <= lon &&
                Grid.lon_low + (lon_ind + 1)*res
}
```

3.13 Coloring Counties based on NDVI Means

Color USA counties by mean NDVI values

Input

- `EOSCUBE database of AVHRR data`
- `EOSCUBE database of USA counties`
- `month`

Output

`Result` of type `C3_Vector<C3_Pair<C3_String,C3_String>>` to hold pairs of county name and the associated color, for each county.

Description

See the description of the USA database of counties in the EOSCUBE Language And Features section

Sample Input

`N/A`

Output For The Sample Input

N/A

Code

```
EXEC COMPREH
SELECT { C3_Pair<C3_String,C3_String>(county.name, Color(ndvi_val))}
INTO { C3_Vector<C3_Pair<C3_String,C3_String>> } Result
FROM { all_counties } AS (County} county
      SELECT { cell.ndvi } INTO { C3_Avg<double>} ndvi_val
      DEFINE area_mbr AS { county.extent.Get_MBR() }
      FROM {Grid.Lat_Ind_Range(area_mbr ) AS {C3_Int32} lat_ind
      FROM {Grid.Lon_Ind_Range(area_mbr ) AS {C3_Int32} lon_ind
         DEFINE delta AS { (Grid.Resolution)/2.0 }
         DEFINE center_lat AS { Grid.Low_Lat+(Grid.Resolution)*lat_ind + delt
         DEFINE center_lon AS { Grid.Low_Lon+(Grid.Resolution)*lon_ind + delta
         DEFINE square AS { (center_lat - delta <= lat <=  center_lat + delta)
                     && (center_lon - delta <= lon <=  center_lon + delta)}
      WHERE { area.TV( lat/lat_val, lon/lon_val) }
         DEFINE cell AS { Grid[lat_ind][lon_ind][month] }
      WHERE { cell.NDVI >= 0 }
      ;
;
END COMPREH

C3_String Color( C3_Double value )
{
if ( (value <= 0.3 ) )
{
return "green";
}
else if (  (value <= 0.5 ) && (value >= 0.3 ) )
{
  return "yellow";
}
else if (  (value >= 0.5 ) )
{
  return "red";
}
};
```

3.14 Counties and NDVI Coverage

Find the counties where areas of NDVI coverage of `ndvi_lwb` are at least the given `percentage` of the county's total area

Input

- `EOSCUBE database of AVHRR data`
- `EOSCUBE database of USA counties`
- `month`
- `ndvi_lwb`
- `percentage`

Output

`Result` of type `C3_Vector<C3_String>>` to hold county names.

Description

straightforward

Sample Input

`N/A`

Output For The Sample Input

N/A

Code

```
EXEC COMPREH
SELECT { square } INTO { C3_DC_RANGE }  ndvi_coverage
FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lat_Ind_Upb,1)} AS {C3_Int32} lat_in
FROM { C3F_Range_Monoid<C3_Int32>(0, Grid.Lon_Ind_Upb,1)} AS {C3_Int32} lon_i
WHERE { Grid[lat_ind][lon_ind][month_1].NDVI >= ndvi_lwb }
        DEFINE delta AS { (Grid.Resolution)/2.0 }
        DEFINE center_lat AS { Grid.Low_Lat+(Grid.Resolution)*lat_ind + delt
        DEFINE center_lon AS { Grid.Low_Lon+(Grid.Resolution)*lon_ind + delta
        DEFINE square AS { (center_lat - delta <= lat <=  center_lat + delta)
                    && (center_lon - delta <= lon <=  center_lon + delta)}
;
END COMPREH

EXEC COMPREH
SELECT { county.Name }
INTO { C3_Vector<C3_String> } Result
FROM { all_counties } AS {County} county
     DEFINE isec AS { county.extent && ndvi_coverage )
WHERE { SQ(isec)/SQ(county.extent) >= percentage }
;
END COMPREH
```

3.15 Sort in Drop Order

Sort the counties in the order of NDVI drop from the period of the first 12 months to the period of the last 12 months.

Input

- `EOSCUBE database of AVHRR data`
- `EOSCUBE database of USA counties`

Output

`Result` of type `Sorted_Vector<C3_String, C3_Double>` to hold county names sorted in the drop order

Description

straightforward

Output For The Sample Input

See Appendix A.

Code

```
EXEC COMPREH
SELECT {C3_Pair<C3_String, C3_Double>(county.Name, avg_drop) }
INTO {Sorted_Vector<C3_String, C3_Double>} Result
FROM {all_counties} AS {County} county
     DEFINE area_mbr AS { county.extent.Get_MBR() }
FROM {Grid.Lat_Ind_Range(area_mbr ) AS {C3_Int32} lat_ind
FROM {Grid.Lon_Ind_Range(area_mbr ) AS {C3_Int32} lon_ind
  DEFINE delta AS { (Grid.Resolution)/2.0 }
         DEFINE center_lat AS { Grid.Low_Lat+(Grid.Resolution)*lat_ind + delta
         DEFINE center_lon AS { Grid.Low_Lon+(Grid.Resolution)*lon_ind + delt
         DEFINE square AS { (center_lat - delta <= lat <=  center_lat + delta)
                     && (center_lon - delta <= lon <=  center_lon + delta)}
WHERE { area.TV( lat/lat_val, lon/lon_val) }
         DEFINE cell_ts AS { Grid[lat_ind][lon_ind] }
     SELECT { cell_ts[month].NDVI - cell_ts[month+Max_Month/2].NDVI }
     INTO {C3_Avg<C3_Double>} avg_drop
     FROM { C3F_Range_Mon(0, Max_Month-1)} AS month
     ;
;
END COMPREH
```

Sample Input

3.16 NDVI Composition Algorithm

In this section we show an EOSCUBE implementation of the Vegetation Composition Algorithm. The main task of the algorithm is to analyze the input stream of pixel measurements and to apply the corresponding VI computation method based on the number of well measured pixels in each composition period. The detailed description of the algorithm can be found in the Vegetation Index ATBD document, and we will not repeat it here.

This EOSCUBE implementation of the algorithm demonstrates the following EOSCUBE benefits:

- the EOSCUBE program reads as a very well-structured high-level procedure; the correspondence between EOSCUBE code and the data flow diagram in the Vegetation Index ATBD document is very clear
- Using the EOSCUBE language FROM clauses and the RANGE monoid for iteration over the stream indices hides all details of loop creation of a traditional programming language, such as checking if the index is in the range or index increment, thus making the program more compact and readable
- Use of built-in COUNT, MAX, and MAXPAIR monoids allows to perform the operations of counting elements, finding minimums and maximums in just one line of code and within the model of the query language. Also each of those monoids incorporates pieces of the code that would otherwise require a separate implementation

We assume that the input stream is represented by a two-dimensional vector `all_pixels`. The first dimension is spatial and corresponds to all pixels, the second is the temporal dimension with daily resolution. The type of the elements is `Pixel_Data` which contains various input parameters required for the VI Composition Algorithm, such as reflectivities, angles and quality control data. The `Pixel_Data` type is shown below:

```
class Pixel_Data {
 float ro_nir;
 float ro_red;
 float theta_v;
 float theta_s;
 float phi_v;
 float phi_s;
 QC qc;
};
```

The following EOSCUBE product implements the main part of the VI Composition Algorithm, following the algorithm description in the Vegetation Index ATBD document:

```
EXEC COMPREH
FROM { C3F_Range_Monoid<C3_Int32>(0,max_pixels,1)} AS {C3_Int32} pix_ind
FROM { C3F_Range_Monoid<C3_Int32>(0,max_days/8,1)} AS {C3_Int32} 8_ind
     DEFINE Good_Pix_Count AS
     SELECT { day_ind } INTO {Count<C3_Int32>}
     FROM { C3F_Range_Monoid<C3_Int32>(8_ind*8, 8_ind*8+7,1)} AS {C3_Int32} d
     WHERE { Is_Good( all_pixels[pix_ind][day_ind] ) }
     DEFINE NDVI_value AS {
          switch ( Good_Pix_Count ) {
              case >= 5 : BRDF(pix_ind, 8_ind)
              case <= 5 && > 0 : Max_NDVI_Angle(pix_ind, 8_ind)
              case 0 : Max_NDVI(pix_ind, 8_ind)
          }
PERFORM Result[pix_ind][day_ind] = NDVI_value
END COMPREH
```

The programs Max_NDVI_Angle(pix_ind,8_ind),Max_NDVI(pix_ind, 8_ind) and BRDF(pix_i 8_ind) are implemented as described in the Vegetation Index ATBD document and specify different NDVI composition methods. Below we give an implementation for the Max_NDVI_Angle(pix_ind,8_ind) and Max_NDVI(pix_ind, 8_ind) programs:

```
DEFINE Max_NDVI_Angle(pix_ind, 8_ind) AS
     DEFINE ndvi_pair AS
     SELECT C3_Pair<all_pixels[pix_ind][day_ind].theta_v,
                    NDVI(all_pixels[pix_ind][day_ind]) >
     INTO {MINPAIR<C3_Double,C3_Double>} ndvi_pair
     FROM RANGE(8_ind*8, 8_ind*8+7) AS {C3_Int32}  day_ind
     MAX( ndvi_pair.1st,  ndvi_pair.2nd)
END

DEFINE Max_NDVI(C3_int32 pix_ind, C3_int32 8_ind) AS
     SELECT NDVI(all_pixels[pix_ind][day_ind])
     INTO {MAX<C3_Double>} Max_NDVI
     FROM RANGE(8_ind*8, 8_ind*8+7) AS {C3_Int32} day_ind
END
```

Here `MINPAIR` is monoid which computes two smallest numbers in a collection and is capable of storing additional information corresponding to those numbers, which is a pair of NDVI values in this case. The NDVI function encodes the formula computing NDVI out from pixel data using one of the formulas described in the Vegetation Index ATBD document.

Part III
EOSCUBE Features and Language

4 Introduction

Constraints provide a flexible and uniform way to conceptually represent diverse data capturing spatio-temporal behavior, complex modeling requirements, partial and incomplete information etc, and have been used in a wide variety of application domains. Constraint databases (CDBs) have recently emerged to deeply integrate data captured by constraints in databases. Although a relatively new realm of research, constraint databases have drawn much attention and increasing interest, mostly in aspects of expressibility and complexity, but also in algorithms and optimization.

Constraint databases are very promising for applications requiring to support large heterogeneous data sets that can be uniformly captured by constraints. This includes (1) engineering design; (2) manufacturing and warehouse support; (3) electronic trade with complex objectives; (4) command and control (such as spatio-temporal data fusion and sensor management [ABK95] and maneuver planning [BVCS93]); (5) distribution logistics; and (6) market analysis.

While many fundamental research questions are yet to be answered, we believe that the area of constraint databases became mature for a reliable research prototype that could serve as a stable platform for experimentation with algorithms and optimization techniques as well as for real-life case studies of a number of promising application domains. We believe that building such a system is a crucial step toward proving the validity of constraint databases as a technology with a significant practical impact.

Motivation and Design Goals

Until now, most of the work on CDBs has been theoretical (see Related Work section). CDB researchers are being challenged by the question whether the CDB technology can really work on real-life, real-size, real-performance applications, or it is just an intellectual toy that will eventually fade away. This parallels, in a sense, to the state of relational databases before the first two prototypes, Ingres and System-R, were developed. Our view is that the viability test for the CDB technology will be the ability to achieve competitive performance and scalability. Therefore, algorithms, data structures and optimization techniques are the most critical issues.

At the beginning of the EOSCUBE work we had two main choices regarding the design objectives: (1) to naively implement a high-level and purely declarative

constraint DB language, such as *LyriC* , focusing on its interface, but ignoring the performance and scalability, or (2) to develop an extensible infrastructure (i.e. an intermediate, optimization-level language, in which evaluation plans can be explicitly expressed) suitable for developing and testing optimization techniques, algorithms and data structures.

The first choice would lead to a much faster and simpler implementation, and, in fact, this is the way most research prototypes are implemented. The second choice, clearly, is considerably more work- and time-intensive, but is essential for our overall objectives. Of course, ideally, we would like to have both a high-level language and a full-scale optimizer in place, but this would not be possible without developing first the optimization infrastructure of (2). This direction was indeed our choice.

The following were our main design principles:

1. In terms of constraint domains and operators, a careful balance between expressiveness and complexity must be achieved. It is easy to fall into a trap of highly expressive constraint domains for almost any imaginable types of data, but with absolutely impractical complexity.

2. The language and the model should be object-oriented, since many object-oriented features are important for the target applications (e.g. [ABK95]).

3. The language should be suitable for explicitly expressing highly optimized evaluation plans (preserving their I/O time and space complexity). It must be flexible enough to support object-oriented optimization (e.g. ENCORE [Zdo89], O2 [ea90], POSTGRES [SRH90]), constraint database optimization (e.g. [BJM93]) and constraint indexing and filtering (e.g. [BW95]).

4. The system should be extensible with respect to (constraint and other) data types, operators and predicates, and special data structures and algorithms.

5. The system should allow easy interaction with an underlying programming language in order to be usable directly by system or application programmers

6. Provided the previous principles are met, the language should ideally be as high-level and easy-to-use as possible.

EOSCUBE Features and Architecture

The EOSCUBE data manipulation language, *Constraint Comprehension Calculus* is an integration of a *constraint calculus* for extensible constraint domains within *monoid comprehensions*, which were suggested as an optimization-level language for object-oriented queries [FM95]. In the following, when no misinterpretation arises, we will

be using the same name EOSCUBE for both the system and the language of the constraint comprehension calculus. [1]

The data model for constraint calculus is adapted from *constraint spatio-temporal* (CST) objects [BK95], that may hold spatio-temporal constraint data, conceptually represented by constraints (i.e. symbolic expressions). In the current version, linear arithmetic constraints (i.e. inequalities and equations) over reals [2] are implemented. New CST objects are constructed using logical connectives, existential quantifiers and variable renaming, within a mutli-typed constraint algebra.[3] The constraint module also provides predicates such as for testing satisfiability, entailment etc, that are used as selecting conditions in hosting monoid comprehension queries.

CST objects possess great modeling power and as such can serve as a *uniform data type* for conceptual representation of heterogeneous data, including spatial and temporal behavior, complex design requirements and partial and incomplete information. Moreover, the constraint calculus operating on CST is a highly *expressive* and *compact* language. For example, just linear arithmetic CSTs and its calculus currently implemented in the system, allow description and powerful manipulation of a wide variety of data, including (1) 2- or 3-D geographic maps; (2) geometric modeling objects for CAD/CAM; (3) fields of vision of sensors; (4) 4-D (3 + 1 for time) trajectories of objects moving in a 3-D space, based on the movement equations; (5) translations of different system of coordinates; and (6) operations research type models such as manufacturing patterns describing interconnections between quantities of manufactured products and resource materials. It is important to note that the conceptual and physical representations of CST objects are orthogonal: while conceptually constraints are viewed as symbolic expressions, the physical representation is typically chosen to facilitate efficient storage and manipulation.

The general framework of the EOSCUBE language is the *monoid comprehensions* language, in which CST objects serve as a special data type, and are implemented as a library of interrelated C++ classes. The data model for the monoid comprehensions is based on the notion of *monoid*, which is a conceptual data type capturing uniformly aggregations, collections, and other types over which one can "iterate". This includes (long) disjunctions and conjunctions of constraints.

The ability to treat disjunctive and conjunctive constraints uniformly as collections is a very important feature of EOSCUBE: it allows to express and implement many constraint operations through nested monoid comprehensions, i.e. in the same language as hosting queries. For example, the satisfiability test of a disjunction of

[1] In fact, the name EOSCUBE was originated from the shorthand C^3 for Constraint Comprehension Calculus.

[2] using finite precision arithmetic

[3] As explained in later sections, users can view the constraint layer as either calculus, or algebra, interchangeably.

conjunctions of linear inequalities is expressed as a monoid comprehension query that iterates over the disjuncts (each being a conjunction), and tests the satisfiability of every conjunction (using the simplex algorithm).

In turn, the ability to express a constraint operation as a sub-query in the hosting query is crucial for what we call *deeply interleaved optimization*: it gives the flexibility to re-shuffle and interleave parts of the constraint algorithm (sub-query) with the hosting query. This re-shuffling can be done by additional global query transformations (discussed in the paper) involving approximations, indexing, re-grouping, pushing cheaper selections earlier, replacing sub-queries with special-purpose algorithms, and so forth. Figuratively speaking, constraint operations are not treated in EOSCUBE as *black boxes* plugged into a query, which would severely restrict optimization opportunities, but rather as *white boxes with black holes.*

The EOSCUBE system architecture supports:

1. Besides CST objects, any data structures expressible in C++.

2. An extensible family of parameterized and possibly nested collection monoids currently including sets, bags, lists, as well as (long) disjunctions and conjunctions of CST objects.

3. An extensible family of aggregation monoids such as *sum*, *count*, *some* and *all.*

4. An extensible family of search structures implemented as parameterized monoids and currently including B-trees, hashing and kD-trees for multidimensional rectangles. Because the search structures are implemented as monoids, they can be used uniformly anywhere in queries where monoids are allowed.

5. An extensible family of special-purpose algorithms, such as the sort-join and the constraint join [BJM93], which are implemented as parameterized monoids. These special algorithms are important for performance because they cannot be matched, in terms of I/O complexity, by standard monoid comprehension algorithms with dynamic buffer management (although many other algorithms, such as the loop join and standard selections, can). Since the special algorithms are expressed as monoids, they can be easily plugged in monoid comprehension queries to replace equivalent sub-queries.

6. Approximation-based filtering, indexing and regrouping based on internal components of nested collection monoids. These features are especially important for achieving deeply interleaved optimization in presence of constraint operations.

7. Orthogonal features inherited from the commercial OODB ObjectStore, including persistence, dynamic buffer management, transaction management, data integrity, crash recovery, version management, and multi-client/multi-server architecture.

The functionality of the EOSCUBE system is a combination of the new EOSCUBE layer and ObjectStore. EOSCUBE as a virtual system (1) inherits "lower" features of ObjectStore, (2) replaces "middle" ObjectStore's features with those of the EOSCUBE layer, and (3) adds "upper" features of the EOSCUBE layer. The implementation of the EOSCUBE layer, in turn, uses ObjectStore and the linear programming package CPLEX. We feel important to note that while EOSCUBE is a research prototype, we believe that it is a scalable system designed to carry out implementations of serious, massive data applications. That is partly due to the use of commercially available components (i.e. ObjectStore and CPLEX).

EOSCUBE, similar to ObjectStore, can be better viewed as a powerful extension of C++ with constraint database features, rather than a full-scale DBMS, and is currently to be used from within a hosting C++ program. As a C++ extension, EOSCUBE uses the native C++ data structures and its type system. In fact, in the current implementation only the monoid comprehensions are pre-compiled, and all the other EOSCUBE features, including the the constraint calculus, are implemented as C++ libraries; hence the native C++ syntax is preserved.

The use of the "dirty" C++ data model, as opposed to "clean" and formally defined models such as of ODMG OQL [ABD+96] or XSQL [KKS92] was our pragmatic choice due to the intended purpose of EOSCUBE: an intermediate optimization-level language, i.e. one in which an optimizer or a programmer can (explicitly) write highly optimized queries, using appropriate order, nesting, special operators (e.g. for the sort join) and built-in optimization primitives. Because of the intended use as an intermediate language, we prefer to regain the flexibility of and the uniformity with the underlying programming language, C++. We designed EOSCUBE to be used both for the implementation and optimization of high-level constraint object-oriented query languages such as *LyriC* or constraint extensions of OQL, and for directly building software systems (by application or system programmers) requiring extensible use of constraint database features.

The focal point of our work is achieving the right balance between the expressiveness, complexity and representation usefulness [Bro] without which the practical use of the system would not be possible. To that end, the EOSCUBE constraint calculus guarantees polynomial data complexity, and, furthermore, is tightly integrated with the monoid comprehensions to allow deeply interleaved global optimization.

5 CST Objects and EOSCUBE Queries by Example

In this section we informally discuss EOSCUBE queries, including CST objects, the constraint calculus and monoid comprehensions using an Earth image example. We assume, briefly, that a database stores a collection of country(or county) maps, which have extents (or shapes). It also stores an Earth image which is an 1x1 degree resolution grid of pixels. We assume that there are parameter values (like vegetation index, cloud amount, or see surface temperature) associated with each pixel. A scientist then may ask queries that output spatial and temporal descriptions of areas where the parameters satisfy a certain condition. Those queries can be compactly answered in EOSCUBE without using user implemented predicates or functions.

5.1 Constraints, CST objects and Schema by Example

Consider application schema for our earth image example(see Appendix B). The schema uses regular object-oriented features, and also what we call *spatio-temporal constraint* (CST) classes, such as CST(lon, lat), which means, intuitively, a constraint in free variables lon and lat. Consider also a county which is approximated by a 2D conjunctive constraint in the latitude-longitude linear system of coordinates lat, lon. For example, a constraint (formula)

$$(-4 \leq lat \leq 4) \wedge (-2 \leq lon \leq 2)$$

with the variables ranging over reals can be viewed as a set of points

$$\{(lat, lon) | (-4 \leq lat \leq 4) \wedge (-2 \leq lon \leq 2)\}$$

in two-dimensional space and describes, say, the extent of a small rectangular area. More accurately, the constraint formula $(-4 \leq lat \leq 4) \wedge (-2 \leq lon \leq 2)$ will be interpreted as an infinite relation over the schema lat, lon, that contains all tuples (lat, lon) satisfying the constraint.

In the EOSCUBE syntax the above formula will look as follows:

```
(-4 <= lat <= 4) && (-2 <= lon <= 2)
```

We use && and == instead of $\wedge$ and =, correspondingly, to preserve the C++ style. Interestingly, the above constraint syntax is native in C++, which is achieved by

exploiting the C++ operators' overloading mechanism. How this is done is explained in more detail in Section 7.3. Users can intuitively think of a constraint with d free variables as a (possibly) infinite relation of d-tuples, as an object in d-dimensional space (i.e. a set of points), or as a symbolic expression, interchangeably, depending on the application and the context of its use. Thus, we will be referring to a constraint by a generic name CST (i.e. *constraint spatio-temporal*) object.

5.2 EOSCUBE Queries by Example

Consider the following EOSCUBE query, yet without CST objects, which finds a bag of all countries colored in red:

```
SELECT c                                    // for file cabinet
INTO {Bag<COUNTRY*>} result          // result is a bag-collection
FROM all_countries
       AS {COUNTRY*} c             // iterator: fc iterates
                                          //                over BAG
WHERE c->color == ''red''                 // predicate, i.e. condition
```

The SELECT clause is followed by a possibly interleaved list of the FROM-clause iterators and WHERE-clause conditions. Any order of iterators and predicates, in which variables are only used after they are bound is allowed. However, in general, different orders may lead, as we shall see, to different resulting collections. In the SELECT clause we may have any C++ expression, possibly using variables bounded in the iterators, or invoking another monoid comprehension.

The semantics of the query is best understood, intuitively, through the following loop program, which is a conceptual skeleton of the actual algorithm evaluating monoid comprehensions.[4]

```
result = empty_bag;
FOREACH c IN {BAG} all_countries DO
   IF c->color == ''red'' THEN
              INSERT c INTO result
```

Also important to note is that a query can be always written with just one FROM clause containing all the iterators, followed by just one WHERE clause with all the conditions. However, we allow any order of interleaved iterators and conditions, in

[4]The real algorithm also deals with many other issues such as persistence, dynamic buffer management, type management and interface with C++ etc.

order to control the evaluation of the query, as is necessary for optimization-level languages such as EOSCUBE.

Consider an example of an earth area where its extent in latitude and longitude coordinates is the set of points $\{(lon, lat)|(100 \leq lat \leq 110) \wedge (50 \leq lon \leq 60)\}$ which is captured as the CST object $(100 \leq lat \leq 110) \wedge (50 \leq lon \leq 60)$ in free variables lat, lon, i.e., of class CST(lat, lon).

The following EOSCUBE query, for a given area, computes the intersection of this area with each country, then finds VI for each intersection using the mean average, and produces a collection of pairs containing the computed value and the corresponding intersection scaled down by 2 on all coordinates. This query also shows that constraints are used in EOSCUBE queries to manipulate, as well as express boolean conditions on CST objects:

```
CST area = ( 100 <= lat <= 110  &&  50 <= lon <= 60 );

SELECT pair( avg, isec.Subst(lon/lon*0.5, lat/lat*0.5) )
INTO {Bag<pair>} result
FROM all_countries AS c
DEFINE isec AS {CST} c.extent && area
WHERE SAT(isec)
 SELECT pix.VI INTO {AVG<real>} avg
 FROM Grid AS {PIXEL} pix
 DEFINE rect AS (pix.lat-0.5 <= lat <= pix.lat+0.5) &&
                              (pix.lon-0.5 <= lon <= pix.lon+0.5)
 WHERE SAT(rect && isec)
```

Variable names are considered to be a part of the constraint database schema and are analogous to attribute names in the relational model. In constraint formulas the system joins variables that has the same symbolic name. Of course, there are situations when explicit renaming is required to achieve our goals(for example when making a self-join of two conjunctions). Expressions `DEFINE expr1 AS expr2` cause the replacement `expr1` by `expr2` in the remainder of the comprehension; they are used simply as shortcuts. `SAT` stands for the satisfiability test of the constraint expression inside the parentheses which checks whether `rect` intersects `isec`.

Below we describe in more detail different types of queries and their syntax.

Create-a-result query

```
EXEC COMPREH
```

```
SELECT { cpp_expr_1 }
INTO { cpp_type_1 } cpp_var_1
FROM { cpp_expr_2 } AS { cpp_type_2 } cpp_var_2
DEFINE cpp_var_3 AS { cpp_expr_3 }
PERFORM { cpp_expr_4 }
<select_subquery>
WHERE { cpp_expr_5 }
;
END COMPREH
```

cpp_type_1 is a C++ type of the resulting monoid cpp_var_1, which is allocated by EOSCUBE; cpp_expr_1 must yield the type of the elements of the result; cpp_expr_2 must yield a collection monoid with element type cpp_type_2; cpp_var_3 refers to cpp_expr_3 by textual substitution and can be used in the rest of the query; cpp_expr_4 is any expression to be executed; cpp_expr_5 must yield bool or int types that are used as conditions; ¡select_subquery¿ denotes any CCUBE query that can be inserted into the main query; the semicolon wraps up the SELECT (sub)query

Append-to-result query

Same as the create-a-result query, but the INTO clause omits the monoid type:

```
EXEC COMPREH
SELECT { cpp_expr_1 }
INTO cpp_var_1
FROM { cpp_expr_2 } AS { cpp_type_2 } cpp_var_2
DEFINE cpp_var_3 AS { cpp_expr_3 }
PERFORM { cpp_expr_4 }
<select_subquery>
WHERE { cpp_expr_5 }
;
END COMPREH
```

The resulting variable cpp_var_1 is not created here; it is assumed to have been defined previously;

Subqueries

A subquery may be put inside another query just like another clause

```
EXEC COMPREH
...
SELECT ... // main query
...
  SELECT ... // subquery
  ...
  ; //this semicolumn wraps the inner query
...
; //this semicolumn wraps the outer query
END COMPREH
```

Action queries

An action query does not create or append the result. It just executes C++ statements, indicated in the PERFORM clause. Note, there is no SELECT clause in an action query. Action queries are typically used for modifying collections or printing results.

```
EXEC COMPREH
PERFORM
FROM { cpp_expr_1 } AS { cpp_type } iter_name
PERFORM { cpp_expr_2  }
END COMPREH
```

The first PERFORM statement without parameters indicates to EOSCUBE that it is an action query. The second perform statement executes cpp_expr_2 for each iteration.

Part IV
EOSCUBE Background, Architecture and Implementation

6 EOSCUBE Monoids and Monoid Comprehensions

In this section we describe the syntax, semantics and implementation of the EOSCUBE monoids and monoid comprehensions. The formal counterpart of the EOSCUBE monoid comprehensions is *monoid comprehensions* of [FM95], which is a restricted version of *monoid homomorphisms* [BTBN91, BTS91, BTBW92] written using the syntax of *monad comprehensions* [Wad90], as is done by [BLS+94]. We first review the formal definition of monoids and monoid comprehensions borrowing heavily from [FM95] and [BW95].

6.1 Review of Monoid Comprehensions

```
BAG { c | fc ← all_countries,
          c->color == ''red'' }
```

This is the original monoid comprehension syntax for the first EOSCUBE query in Subsection 5.2. Here, `BAG` indicates the type of the resulting collection (monoid); `fc` to the left of | is what we SELECT; ← is used to denote an *iterator*, i.e. the statement in the FROM clause; and the rest is *predicates*, i.e. logical conditions appearing anywhere in the WHERE clauses. The intuitive meaning is given by the nested loop program in Subsection 5.2.

In addition to collections, we can also compute aggregation functions. For example,

```
SUM { 1 | fc ← all_file_cabinets,
          fc->color == ''red'',
          dr ← fc->drawers,
          dr->color == ''blue'' }
```

will count the number of `file_cabinets` in the result.

More formally, a set of basic data types given, e.g., `int`, `real` and `char`, and a set of type constructors, e.g., `set`, `list`, `bag`. A *data type* is defined recursively as a basic data type or a constructed type $T(\alpha)$ determined by the type parameter α.

A *monoid* is a triple $(T, \mathtt{zero}, \mathtt{merge})$, where T is a data type and merge is an associative function, of type $T \times T \rightarrow T$, with left and right identity zero. For example, $sum = (int, 0, +)$ is a monoid. A *collection monoid* is a quadruple $(T(\alpha), \mathtt{zero}, \mathtt{unit}, \mathtt{merge})$, where (1) $T(\alpha)$ is a constructed type determined by the type parameter α, (2) $(T(\alpha), \mathtt{zero}, \mathtt{merge})$ is a monoid, and (3) unit is a function of type $\alpha \rightarrow T(\alpha)$. As an example, $(list(int), [], f, ++)$, where [] is the empty list, $f(i) = [i]$ for each i and $++$ is the concatenation operation on lists.[5] Finally, a *primitive monoid* is a quadruple $(T, \mathtt{zero}, \mathtt{unit}, \mathtt{merge})$, where $(T, \mathtt{zero}, \mathtt{merge})$ is a monoid and unit is the identity function of type $T \rightarrow T$. Examples of primitive monoids include $prod = (int, 1, id, *)$, where $id(i) = i$ for each integer.

Intuitively, a monoid $\mathcal{M} = (T, \mathtt{zero}, \mathtt{merge})$ is an abstract definition of a data type. Collection monoids capture the bulk types, and primitive monoids capture the basic types. Each instance of the collection type $\mathcal{M} = (T(\alpha), \mathtt{zero}, \mathtt{unit}, \mathtt{merger})$ is expressed as compositions of functions zero, unit and merge on instances of type α. As an example, the monoid $(list(int), [], f, ++)$ given earlier defines a data type of the integer lists. An instance of the type is intuitively a list of integers and the list is expressed as a composition of functions [], u and $++$ applying on integers. For example, the list $\{1, 2, 3, 1\}$ can be expressed as $++(u(1), ++(u(2), ++(u(3), ++(u(1), [])))$.

A monoid $(T, \mathtt{zero}, \mathtt{merge})$ is called *commutative* (*idempotent*, resp.) if function merge is commutative (idempotent, resp.). For example, the monoid $set^{\alpha} = (set(\alpha), \{\}, f', \cup\}$, where $f'(i) = \{i\}$ for each instance i of type α, is a commutative and idempotent monoid, and $bag^{\beta} = (bag(\beta), \{\!\{\ \}\!\}, f'', \hat{\cup})$, where $f''(i) = \{\!\{i\}\!\}$ for each instance i of type β and $\hat{\cup}$ is the additive bag union, is a commutative monoid. Intuitively, less properties correspond to more structure. For example, the monoid *bag* has more structure than the monoid *set* because repetitions in a *bag* do matter (since it is not idempotent), whereas in a *set* do not (since it is idempotent). Similarly, the monoid *list*, being not commutative, has more structure then the monoid *bag*, which is commutative. For monoids $\mathcal{M}$ and $\mathcal{N}$, we say $\mathcal{N} \preceq \mathcal{M}$ if that $\mathcal{N}$ is commutative (idempotent, resp.) implies that $\mathcal{M}$ is commutative (idempotent, resp.), i.e. $\mathcal{N}$ has the same or less properties than $\mathcal{M}$. This exactly corresponds to the intuitive notion that $\mathcal{N}$ has more structure than $\mathcal{M}$. It is easily seen that $bag^{\beta} \preceq set^{\alpha}$. If $\mathcal{N} \preceq \mathcal{M}$, then an instance of type $\mathcal{N}$ can be "translated" deterministically, by using the merge function of the monoid $\mathcal{M}$, into an instance of the type $\mathcal{M}$, but not necessarily vice versa.

Queries on monoids are expressed as monoid comprehensions. A *monoid comprehension* over the monoid $\mathcal{M}$ takes the form

$$\mathcal{M}\{e \mid r_1, \ldots, r_n\}$$

[5] We use $[a_1, \ldots, a_n]$ to denote a list and $\{\!\{a_1, \ldots, a_k\}\!\}$ to denote a bag.

where e is an expression called the *head* of the comprehension, and $r_1, \ldots, r_n$ is a list of *qualifiers*, each of which is either

- a *iterator* of the form $v \leftarrow e'$, where v is a variable, and e' is an expression that evaluates to an instance of a collection monoid of type which is $\preceq \mathcal{M}$ or
- a *selection-predicate*, which is an expression that evaluates to `true` or `false`.

The expressions in turn can include monoid comprehensions. An important condition for the monoid comprehension is that for each $1 < i \leq n$, each free variables (i.e., free variables in the expressions and predicates) appearing in $r_i, \ldots, r_n$ must appear as the variable of an iterator among $r_1, \ldots, r_{i-1}$, and each free variables in the e must appear as the variable of a iterator among $r_1, \ldots, r_n$.

It is assumed that each instance of a monoid appearing as argument in a monoid comprehension is represented as an expression involving `merge`, `unit` and `zero` functions. For example $\texttt{BAG}\{1, 2, 1, 3\}$ can be represented as

```
merge( merge(unit(1),unit(2)),
       merge(unit(1),unit(3)))
```

or, since `merge` is associative and `zero` is a left (and right) identity, as

```
merge( merge( merge( merge( zero,
                            unit(1)),
                     unit(2)),
              unit(1)),
       unit(3))
```

We will assume that every monoid instance is (conceptually) represented this way, and, thus, the notation $\mathcal{N}\{a_1, a_2, \ldots, a_n\}$ will denote the expression

$$\texttt{merge(...\ merge(merge(zero,unit(}a_1\texttt{)),unit(}a_2\texttt{)),\ ...,unit(}a_n\texttt{))}$$

Furthermore, $\mathcal{N}\{\}$, and $\mathcal{N}\{a_1, a_2, \ldots, a_n\}$ where $n = 0$ will both denote $\texttt{zero}^{\mathcal{N}}$, i.e. the empty monoid instance.

A monoid comprehension `M` over a monoid $\mathcal{M} = (T, \texttt{zero}, \texttt{unit}, \texttt{merge})$(collection or primitive), defines an instance of type T [6] by first initializing `result` with $\texttt{zero}^{\mathcal{M}}$, and then invoking the procedure `insert_MC(result,M)` defined recursively by the following reduction rules:

[6] Our definition here is different from, but equivalent to the original one; ours is closer to the implementation.

(r1) insert_MC(result, $\mathcal{M}\{e\,|\}$)
$\rightarrow$ result := merge$^{\mathcal{M}}$(result, unit$^{\mathcal{M}}(e)$)

(r2) insert_MC(result, $\mathcal{M}\{e \mid \text{false}, \vec{r}\}$)
$\rightarrow$ nil (i.e. do nothing)

(r3) insert_MC(result, $\mathcal{M}\{e \mid \text{true}, \vec{r}\}$)
$\rightarrow$ insert_MC(result, $\mathcal{M}\{e \mid \vec{r}\}$)

(r4) insert_MC(result, $\mathcal{M}\{e \mid x \leftarrow \mathcal{N}\{a_1, \ldots, a_n\}, \vec{r}\}$)
$\rightarrow$ for $i = 1$ to n do insert_MC(result, $\mathcal{M}\{e \mid \vec{r}\}[x/a_i]$)

where $\mathcal{N}$ is a collection monoid $(S, \text{zero}^{\mathcal{N}}, \text{unit}^{\mathcal{N}}, \text{merge}^{\mathcal{N}})$ with the condition $\mathcal{N} \preceq \mathcal{M}$. Note that $\mathcal{M}\{e \mid \vec{r}\}[x/a_i]$ denotes the replacement of x with a_i in $\mathcal{M}\{e \mid \vec{r}\}$. Note also that the last rule is deterministic as far as the resulting monoid instance is concerned.

6.2 Monoids in EOSCUBE

To understand the minimum requirements for primitive and collection monoids, consider the recursive rules defining the result of a monoid comprehension. For the monoid $\mathcal{M}$ to appear in the result of the monoid comprehension, we only need to (1) use zero$^{\mathcal{M}}$ and (2) know how to perform

result = merge$^{\mathcal{M}}$(result, unit$^{\mathcal{M}}(e)$)

which is, in fact the insert$^{\mathcal{M}}$(result, e) operation (i.e. we define insert$^{\mathcal{M}}$ this way). [7] In order for the collection monoid $\mathcal{N}$ to appear inside the comprehension, we only need to be able to *iterate* over $\mathcal{N}\{a_1, \ldots, a_n\}$, i.e. to perform the **for** loop.

The representation (and implementation) of collection monoids in EOSCUBE is based on two C++ template classes, parameterized with the type A of collection elements: CollectionMonoid and Iterator:

```
template < class A > class CollectionMonoid
{
friend class Iterator<A>;
public:
    CollectionMonoid();              //  C++ constructor used as zero
    virtual void Insert( A& ) = 0;
    virtual Iterator<A>* CreateIterator() = 0;
private:                             //  specific subclasses contain
```

[7] For a primitive monoid the name insert is probably strange; we really mean by insert exactly result := merge(result, unit$^{\mathcal{M}}(e)$). For example, for primitive monoid sum $(int, +, 0, \text{identity})$, insert$^{\mathcal{M}}$(result,5) is result:=result+ 5.

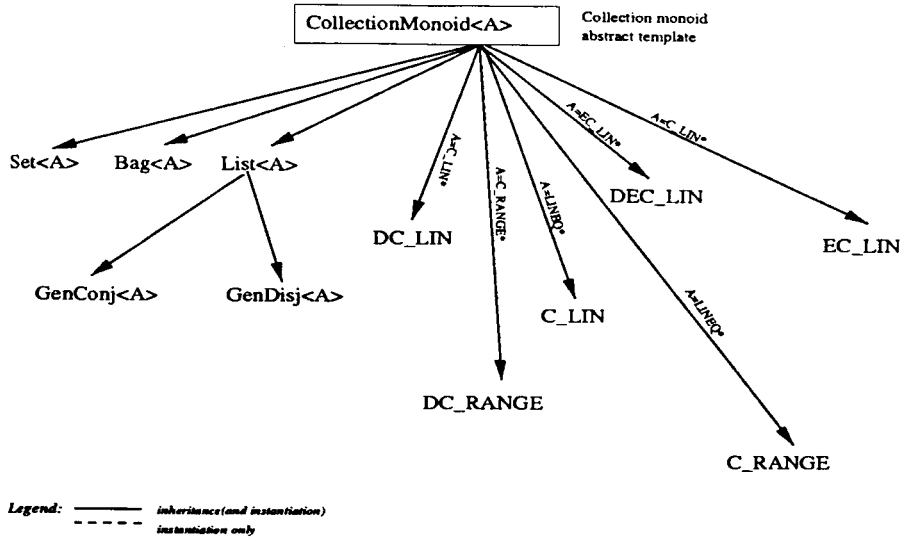

Figure 2: Collection Monoids in EOSCUBE

```
                                        //  actual implementation
};
```

The class `CollectionMonoid` reflects the minimum requirements: it has `zero`, implemented as a class constructor, and `Insert` and `CreateIterator` member functions. An `Iterator` object, created by `CreateIterator`, has `First`, `More` and `Next` member functions which can be directly used in the C++ `for`-loop. Specific collection monoids implemented in EOSCUBE, depicted in Figure 4, are implemented each with two classes derived from the classes `CollectionMonoid` and `Iterator`, correspondingly. The collection monoids `list`, `set`, and `bag` are currently implemented using the ObjectStore collections.

As opposed to collection monoids, primitive ones only require `zero` and the `insert` member function, since they are not used in the query iterators.

```
template< class T > class PrimitiveMonoid
{
public:
  PrimitiveMonoid( );                  // Note: creates zero of monoid
  virtual void Insert( T& ) = 0;
  operator T ();
  static T zero;
protected:
  T value;
```

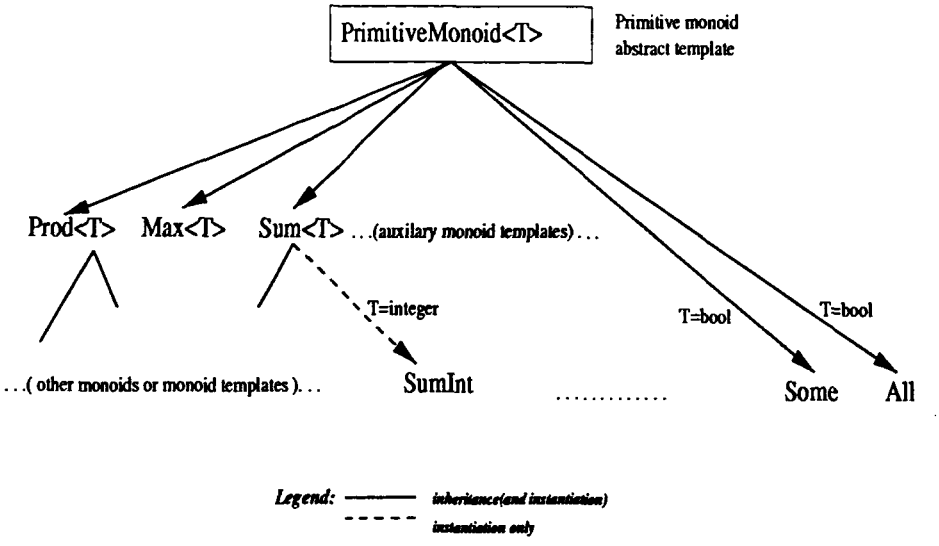

Figure 3: Primitive Monoids in EOSCUBE

```
};
```

The extensible family of primitive monoids and monoid templates in EOSCUBE, depicted in Figure 5, includes `Prod<T>`, `Max<T>`, `Sum<T>`, `Some` and `All`. Note, that the type used in `Some` and `All` is `bool`. These monoids work as disjunction and conjunction of conditions, respectively.

6.3 Syntax and Semantics of EOSCUBE queries

The syntax of the EOSCUBE comprehensions has been explained by examples. More accurately, it is of the form:

```
   SELECT C++expr
   INTO [{monoid_type}] [result]
%   [from-where-define-list]
   [from-where-define-perform-list]
```

`C++expr` in the SELECT clause is an arbitrary C++ expression that evaluates to the type of `result's` elements. Note that `C++expr` may involve variables instantiated in the FROM clauses and may also contain nested monoid comprehensions. The first (optional) parameter in the INTO clause specifies the type of `result`. If this argument is omitted, the system assumes that `result` is defined elsewhere in the C++ program. When the monoid comprehension is nested, `result` argument may

be omitted. The `from-where-define-perform-list` is a sequence of the FROM, DEFINE, PERFORM and WHERE clauses (explained earlier by examples) in any order. Note that any number of iterators, separated by commas, may appear in each FROM clause; further, any number of predicates (conditions) may appear in each WHERE clause. Also important is that nesting is recursively allowed anywhere in the monoid comprehension, provided that nested monoid comprehensions return appropriate types. For instance, collection monoid comprehensions may stand anywhere a collection monoid can; or, monoid comprehensions returning TRUE or FALSE may stand in place of any predicate. This flexibility also enables the use of special algorithms (such as for the constraint and sort join, indexing or regrouping), provided that they produce the appropriate types (e.g. collection monoid) as their outputs.

The semantics of EOSCUBE monoid comprehension queries is defined by the corresponding formal monoid comprehension. Furthermore, the basic evaluation is by the nested loop algorithm with dynamic buffer management. Important, however, is that the nested monoid comprehension in the FROM clause does not create physical intermediate results, but rather supports the pipe-lining.

7 CST Objects and Constraint Calculus

7.1 Framework for Constraint Algebra and Calculus

EOSCUBE uses the multi-typed algebra framework of [Bro], which we review here. As seen in the examples, the notion of CST data relies on a simple and fundamental duality: a constraint (formula) ϕ in free variables $x_1, \ldots, x_n$ is interpreted as a set of tuples $(a_1, \ldots, a_n)$ over the schema $x_1, \ldots, x_n$ that satisfy ϕ; and, conversely, a finitely representable object in $(x_1, \ldots, x_n)$ space can be viewed as a constraint. That is, the syntax is constraints, i.e. symbolic expressions; the semantics are the corresponding, possibly infinite, relations.

CST objects are represented by a sub-family of the first order logic, (i.e. with the logical connectors $\wedge$, $\vee$, $\neg$ and $\exists$) and by a family of atomic constraints, such as linear arithmetic over reals, polynomial or dense order. CST objects are manipulated by means of a *constraint algebra*, whose operators are expressed using a sub-family of the first-order logic, renaming of variables, and atomic (e.g. arithmetic) constraints. For example, if P and Q are CST objects in $x_1, \ldots, x_n$, their intersection can be represented by $P \wedge Q$; union by $P \vee Q$; the test of containment of P in Q by $\forall x_1, \ldots, \forall x_n(P \longrightarrow Q)$ (this is, in fact, the entailment test, ENTAIL); emptiness of P by $\neg\exists x_1, \ldots, \exists x_n P$ (this is, in fact the satisfiability test, SAT); disjointness of P and Q by $\neg\exists x_1, \ldots, \exists x_n(P \wedge Q)$; the projection of P on axes $x_1, \ldots, x_i$, $1 \leq i < n$, by $\exists x_{i+1} \ldots \exists x_n P$ etc. If we only use linear constraints over reals, as implemented in

EOSCUBE, within the first-order logic we can express any linear transformation such as rotation, translation and stretch; check convexity, discreteness and boundness [?]; compute the convex hull, augment objects, change coordinate systems; etc.

Thus, constraint objects can be manipulated by a very *expressive language.* Moreover, since this language uses only a small number of operators (i.e. logical connectors and quantifiers), it is also very *compact,* as compared to using a separate operator for each specific type of transformation, which is typically done in extensible or spatial database systems. It is also claimed, that for linear constraints, query languages manipulating constraint objects are deeply *optimizable,* in terms of indexing and filtering (e.g. [BLLM95, KRVV93, Sri92]), and constraint algebra algorithms and global optimization (e.g. [BJM93, GK95]).

More specifically, constraint algebras operate on a family $\mathcal{F}$ of canonical representations of constraint expressions (objects). For constraint objects $C_1, \ldots, C_n$ a first-order logic formula $\phi(C_1, \ldots, C_n)$ such as $\exists y(C_1[u_1/y, v_1/z] \wedge \ldots \wedge C_n[u_n/y, v_n/z])$, where $[u_i/y, v_i/z]$ denotes the variable replacement, defines the following constraint algebra operator *op*: (1) replace each C_i by the corresponding constraint expression, (2) do all variable replacements and (3) transform the resulting constraint expression into the required (equivalent) canonical representation in $\mathcal{F}$. Thus *op* can be seen as a function from $\mathcal{F} \times \ldots \times \mathcal{F}$ to $\mathcal{F}$. On the other hand, the operator *op* has the interpretation $\mathcal{I}(op)$, which is a query that maps n relations to one. Given $\mathcal{I}(C_1), \ldots, \mathcal{I}(C_n)$, where $\mathcal{I}(C_i)$, $1 \leq i \leq n$, is the relational interpretation of C_i and $x_1, \ldots, x_m$ are all free variables, $\mathcal{I}(op)$ computes the following relation:

$$\{(x_1, ..., x_m) \mid \phi(C_1, \ldots, C_n)\}$$

Clearly, the duality between constraints and point sets carries over to the constraint algebra/calculus, that is, the following commutative property holds:

$$\mathcal{I}(op(C_1, \ldots, C_n)) = \mathcal{I}(op)(\mathcal{I}(C_1), \ldots, \mathcal{I}(C_n))$$

A constraint family $\mathcal{F}$ is defined by choosing (1) an atomic constraint domain, (e.g. polynomial over reals or linear over integers), (2) the structure of the logical formula allowed (e.g. disjunction of conjunctions or existentially quantified disjunction) and (3) the required canonical form (e.g. whether to eliminate existential quantifiers, eliminate each redundant disjunct, extract all implicit equalities in conjunction or eliminate redundancy in conjunctions). The definition of a constraint algebra amounts to choosing the structure of first-order formulae and the atomic constraints allowed in the query.

The challenge here (and a major area of research) is the development of constraint families and algebras, that strike, for each application realm, a careful balance between (1) *expressiveness,* (2) *computational* complexity and, very importantly, (3) *representation usefulness.*

As one extreme, if the entire first-order logic (as studied in [?, ?]), and the same atomic constraints are allowed in both the constraint family $\mathcal{F}$ and the algebra, we get a very expressive algebra with the low data complexity, since no actual manipulation of constraints would be required. However, the representation of the result might consist of a very large unsimplified constraint expression that might not be useful for the user. For instance, the answer to a query "is constraint object C empty" would be $\exists x_1 \ldots \exists x_n C$, where $x_1, \ldots, x_n$ are all free variables, whereas the user expects a `true` or `false` answer.

An example of a very expressive, but having high (exponential) time data complexity is the DISCO (Datalog with Integer and Set order COnstrains) query language [BR95]. Constraint representation is DISCO is useful in many, but not all applications. For example, to express satisfiability of a simple propositional formula, the user needs to encode the formula by a datalog (with constraints) program, in a fairly unnatural way.

Close to the other end, the framework [?] requires a fairly restricted sub-family of first-order logic in constraint objects: disjunction of (unquantified) conjunctions of atomic constraints (the algebra, however, allows more, including quantifier elimination). This representation is useful for many, but not all applications: for example a constraint representation of a triangle given by vertices $(a_1, b_1), (a_2, b_2), (a_3, b_3)$,

$$\exists t_1 \exists t_2 \exists t_3 (x = a_1 t_1 + a_2 t_2 + a_3 t_3 \wedge y = b_1 t_1 + b_2 t_2 + b_3 t_3 \wedge t_1, t_2, t_3 \geq 0 \wedge t_1 + t_2 + t_3 \leq 1)$$

is not directly representable in that framework. Still, for some atomic constraint families, such as linear inequalities over reals, this framework may be computationally unmanageable: the quantifier elimination may result in a constraint exponential in the size of the original conjunction, although for many sub-families more efficient algorithms were developed (e.g. [GK95, JMSY92, HLL90, LL91]). A more flexible first-order logic structure that allows the entire linear constraints over reals while controlling computational complexity was described in [BJM93, BK95].

7.2 EOSCUBE Constraint Families and Canonical Forms

In EOSCUBE we concentrate on linear constraint over reals, which are expressive and useful in a variety of application domains. However, in order to control the computational complexity, we design a more flexible first-order logic structure by constructing a number of interrelated constraint families. This continues the line of work in [BJM93, BK95].

The six interrelated constraint families in EOSCUBE are depicted in Figure 6. The four main families are for unrestricted linear constraints over reals: `C_LIN`, for

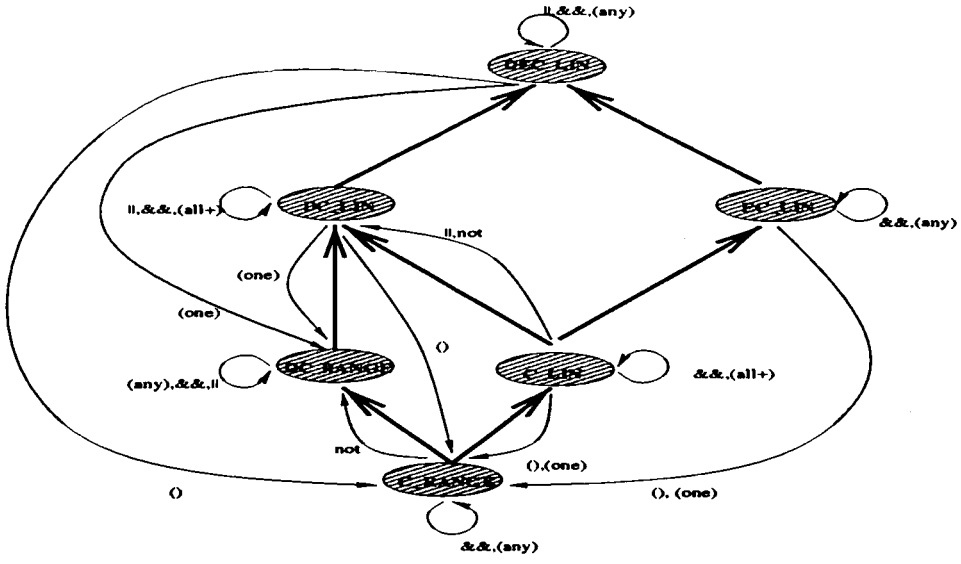

Figure 4: Families of CST Objects

Conjunctive Linear, stands for constraints represented in the form $\wedge_{i=1}^{n} C_i$, where C_i is a linear inequality; **EC_LIN**, for Existential Conjunctive, corresponds to the form $\exists \vec{x} \wedge_{i=1}^{n} C_i$; **DC_LIN**, for Disjunctions of Conjunctions, corresponds to the form $\vee_{i=1}^{m} \wedge_{j=1}^{n} C_{ij}$; and **DEC_LIN**, for Disjunctions of Existential Conjunctive, corresponds to the form $\exists \vec{x} \vee_{i=1}^{m} \wedge_{j=1}^{n} C_{ij}$. The other two families are for range constraints, i.e. of the form a op x op b, where op is either $<$ or $\leq$ and a and b are either real numbers or $-\infty$ or ∞. Namely, **C_RANGE**, for Conjunctive Range, stands for constraints represented in the form $\wedge_{i=1}^{n} C_i$, where C_i is a range constraint; and **DC_RANGE** corresponds to the form $\vee_{i=1}^{m} \wedge_{j=1}^{n} C_{ij}$.

We use the EOSCUBE notation for operations: `not`, `&&`, `||`, and `(...)` for projection. We distinguish between projections on one variable, denoted `(one)`; on zero attributes, denoted `()`, i.e. all free variables are existentially quantified; on all variables, denoted `(all+)`, i.e. no variables are quantified; and, on any number of attributes, denoted `(any)`, for arbitrary projection. The user is recommended to use the most specific projection operator in order to achieve the strongest (i.e. lowest) resulting types.

Not only the projections in EOSCUBE can eliminate existing free variables, but they can also add new ones. For example, a CST `(1 <= x <= 5)` can be transformed by the "projection" on `(x,y)` into `(x,y) | (1 <= x <= 5)`, thus adding the new free variable `y`, and getting a new interpretation as a relation over `x,y` of all tuples with `x` as required and an arbitrary real number `y`. However, in the classification of the projection cases discussed earlier, we only consider free variables physically appearing in the constraint expressions.

Thick arrows indicate type hierarchy. For example, `C_LIN` is a sub-type of `DC_LIN`, `EC_LIN` and, transitively, `DEC_LIN`, meaning that a CST object of type `C_LIN` may be used as an argument wherever its supertypes are allowed.

Thin arrows indicate, for each constraint family, the allowed operations and the type of the result, which may belong to a different constraint family. For example, `&&` is allowed on `C_LIN`, returning the result in the same family, while `()`, and `(one)` return the result in `C_RANGE` (which is also in `C_LIN` as a supertype of `C_RANGE`). Note, that the result of `||` on arguments from `C_LIN` will be in `DC_LIN`, not in `C_LIN`. Some of the operations are implicit: for instance, while `||` does not explicitly appear in `C_LIN`, it can be applied since it is allowed for its supertype `DC_LIN`.

Operators may be overloaded: for example `&&` in `C_LIN` is different from `&&` in `DC_LIN`; they are implemented differently and return the results of different types (with different representations). The actual operator applied depends on the types of its arguments. As an example, `&&(C_LIN,C_LIN)` will use the `C_LIN` operator; whereas, `&&(DC_LIN,DC_LIN)`, as well as `&&(C_LIN,DC_LIN)` will use the operator from `DC_LIN`. In general, for the application of `op(arg1,arg2)` we use the lattice structure of the type hierarchy, where $\texttt{sub-type} \preceq \texttt{super-type}$ (i.e. the higher the bigger). The actual `op` chosen is the one of the CST type that is the `least upper bound` of `type(arg1)` and `type(arg2)` on which `op` is defined. Note, that the CST families are constructed in such a way that, for every `op` used, such least upper bound, if exists, is unique; hence, there is no ambiguity. If no such bound exists, `op` is not allowed on `arg1` and `arg2` and would result in a compile-time error.

For users of the EOSCUBE CST library it is easy to remember what's allowed and what's not. `&&`, `||`, and `()` can be freely applied on arguments of any CST family. Only `not` is restricted: it can only be applied to arguments of the type `C_LIN` (and thus its sub-type `C_RANGE`). The system will always produce the strongest (i.e. least) type possible for the resulting constraint.

In addition to the logical algebraic operators, all families have the following operators:

1. `RENAME(CST_obj,''x1/e1,...,xN/eN'')` where `x1,...,xN` are the names of the variables to be replaced with the variables `e1,...,eN`.

2. `SAT(CST_obj)` to check satisfiability of the argument, i.e. whether there exists

an assignment of real constants into its free variables that makes it true. There is also `MUT_SAT(CST_obj_1, CST_obj_2)` which is equivalent to `SAT(CST_obj_1 && CST_obj_2)`.

3. `TRUTH_VALUE(Var_Assign,CST_obj)` returns the truth value of the CST under the assignment `Var_Assign` of constants into the `CST_obj` free variables.

4. `MIN_POINT(lin_func,CST_obj)` and `MAX_POINT(lin_func,CST_obj)` where `lin_func` is a linear function with real coefficients. Returned is the assignment of constants into the variables of `lin_func` that maximizes it subject to the constraints in the CST.

5. `MIN(var_name,CST_obj)` and `MAX(var_name,CST_obj)` that return MIN and MAX of the first argument subject to the constraints in the second.

Note that the MIN and MAX operators correspond to the problem of linear programming. In addition, `ENTAIL(DC_LIN,C_LIN)` operator is allowed[8].

Finally, since all the disjunctive CST objects can be considered as collections of disjuncts and all the conjunctive CST objects as collection of conjuncts, we make these CST families EOSCUBE collection monoids by implementing the required iterators and member functions.

The six CST families are carefully constructed with the complexity consideration in mind as follows. First, all operations allowed on the families have polynomial data complexity. This is the reason, for example, that `C_LIN` is not closed under the general projection: transforming the result into `C_LIN` will require quantifier elimination and thus the size of the result (and, of course, time complexity) may be exponential in the number of variables eliminated. Whereas, `EC_LIN` is closed under the general projection since the general projection in `EC_LIN` is *lazy*: `EC_LIN` allows quantifiers in the internal representation and hence no physical quantifier elimination is performed. Similar, `not` is allowed on conjunctive CST families, `C_RANGE` and `C_LIN`, but not on, say, `DC_LIN`. The reason is that transforming an expression of the form $\neg \vee \wedge C$ or, of the form $\wedge \vee \neg C$, into `DC_LIN` may result in an expression of an exponential size, which we would like to avoid. We discuss what operators involve computationally in more detail in the next subsection.

The CST families use canonical forms, i.e. useful standard forms of constraints, that we adopt in EOSCUBE from [LHM89, BJM93] and review here from [BJM93]. For CST objects in the disjunctive families, some disjuncts might be redundant in the sense that omitting them results in an equivalent constraint. Clearly, a canonical form that eliminates such disjuncts would be desirable. However, the problem of detecting

[8] and, of course, for all subtypes of `C_LIN` and `DC_LIN`

such tuples is co-NP-complete [Sri92], and so we will perform only one simplification of disjunctions: the deletion of inconsistent disjuncts.

For CST objects in the conjunctive families, there are a number of simplifications that can be requested by the user. One choice is to write all the equations in the form $\{x_i = t_i \mid i = 1, \ldots, n\}$ where x_i's are distinct and appear nowhere else in the constraint. A second choice is whether all equations which are implicit in the inequality constraints should be represented explicitly. (As a simple example of this, consider the constraints $x + y \leq 2, x + y \geq 2$.) A third is the extent to which redundancy within the inequalities should be removed. [LHM89] presents a classification of redundancy that suggests simple forms of redundancy removal. A fourth choice is whether to keep the inequalities in a different form, such as the simplex tableau form. In the current EOSCUBE implementation, the only simplification is the removal of inconsistent disjuncts in the disjunctive families; however, a range of simplifications on the conjunctions is presently being implemented.

7.3 Implementation of CST families

On the conjunctive families, the `&&` operator simply combines its arguments and is constant time; `(one)|C`, which is a projection on a single variable, involves applying a linear program (using the simplex algorithm of CPLEX) twice for finding the minimum and the maximum of the variable subject to `C`; `()|C`, which is eliminating all variables in `C` works as a satisfiability test, using the first phase of the simplex, as does the `SAT` predicate.

On the disjunctive families, the `||` operator is constant-time, while `D1 && D2`, where $D1 = \vee_{i=1\ldots n} C1_i$ and $D2 = \vee_{j=1\ldots m} C2_j$ is more involved:

$$D1 \wedge D2 = \bigvee_{i=1,\ldots,n} C1_i \wedge \bigvee_{i=1\ldots m} C2_j = \bigvee_{i=1\ldots n, j=1\ldots m} (C1_i \wedge C2_j)$$

that is, the result consists of all combinations of $C1_i$ and $C2_j$ that are mutually consistent (i.e. their conjunction is satisfiable). Since the CST families are monoids, `D1 && D2` is implemented as the following EOSCUBE query:

```
SELECT *c1 && *c2
INTO  {DEC_LIN} conj_D1_and_D2
FROM  D1 AS {EC_LIN*} c1,
      D2 AS {EC_LIN*} c2
WHERE SAT(*c1,*c2)
```

We show how such queries are optimized using the approximation-based filtering, indexing and re-groupings in the next section. Similar, `SAT(D)`, where `D` is of the type `DEC_LIN` (as well as other disjunctive families) is represented as

```
SELECT SAT(*c)
INTO  {Some} satisf_flag
FROM  D AS {EC_LIN*} c
```

Note, that c is of the type EC_LIN and so SAT in the WHERE clause works on conjunctions. Further recall that Some is a primitive monoid whose merge operator is a logical or; thus, the satisf-flag will be true if and only if at least one component is true. Finally, ENTAIL(D,C), where D is DC_LIN and C is C_LIN, is represented as

```
SELECT ENTAIL(*c,C)
INTO  {Some} imply_flag
FROM  D AS {C_LIN*} c
```

Beyond the algorithms for constraint operations discussed, there are two subtle design problems that we address in EOSCUBE: compile-time maintenance of the type lattice and a lazy evaluation. Support for the lazy evaluation of constraint (i.e. involving CST) expressions is necessary for efficiency. For example, if we are interested in the SAT test of an expression involving logical connectors, it is typically wasteful to perform simplifications of subexpressions.

To exemplify the problem arising from the type lattice maintenance, consider the && operator. In fact, while && has one conceptual meaning, it works differently in every CST family. Moreover, since && is defined on DEC_LIN, the arguments may be any subtypes of DEC_LIN. Thus, every ordered pair of (sub) types for arguments of && works uniquely: we need to find the least upper bound type, to perform corresponding type conversions, and then to apply the physical algorithm of the resulting CST family. One possibility is implementing a separate function for each pair of subtypes, but this would result in a quadratic number of functions for each logical operator: 30 for the six families, and impractically many for future extensions of EOSCUBE with new CST families. On the other hand, the direct implementation of a subtype relationship using the C++ inheritance mechanism does not work, since each family has its own implementation, data-structures etc, which should not be inherited by its subclasses. Of course, there is also a possibility of maintaining just one global CST type, and to distinguish individual subfamilies only at run time. This, however, would eliminate the capability of the compile-time type checking, an important feature of EOSCUBE.

To solve the type lattice problem we designed a two-layer architecture for the CST families: the lower layer, called basic_CST, supports the physical representation and manipulation of the CST families; the upper layer, called lazy_typed_CST, is responsible for the type lattice management and the lazy evaluation, while the actual evaluation is passed to the lower, basic_CST layer.

The basic_CST layer is composed of the six classes basic_C_LIN, basic_DC_LIN etc., each maintaining its own data structures to represent the underlying constraints;

and one super (base) class, basic_CST. No automatic type casts are supported on this layer. However, each basic family has member functions for explicit type conversions into basic types that are higher in the type hierarchy. For example, transforming basic_C_LIN into basic_DC_LIN creates a basic_DC_LIN object (disjunction) that has a single disjunct in it.

The lazy_typed_CST layer, on the other hand, does support automatic sub-typing and the ability to determine the least upper bounds of operators' arguments at compile-time. The six families lazy_typed_C_LIN, lazy_typed_DC_LIN etc., are implemented as six classes with a class hierarchy that exactly matches the type hierarchy of the CST families. However, all the lazy_typed classes have similar internal representation, which is inherited from the abstract class lazy_typed_CST. The representation is basically an expression tree (hence "lazy"), with internal nodes storing the constraint operators (such as && or ||) and encoding the strongest type to which the subtree can be converted; the leaves are objects of the lower layer, basic_CST. It is important to emphasize that the CST type checking we do in EOSCUBE heavily uses capabilities of C++ and would be impossible (at compile-time without any precompiling) in languages such as C.

The EOSCUBE system also supports two generic parameterized CST families: Gen_Conj<T> for generic conjunctions and Gen_Disj<T> for generic disjunctions, where T is an arbitrary, possibly complex, CST type. Both are collection monoids and support the TRUTH_VALUE function; further, SAT is supported by Gen_Disj<T> provided it is supported by T. Also ENTAIL(Gen_Disj<T>,T) is defined provided it is defined on T. These operations are represented again with monoid comprehension queries. For example, SAT(D), where D is of the type Gen_Disj<T>, is represented as

```
SELECT SAT(c)
INTO  {Some} satisf_flag
FROM  D AS {T} c
```

Finally, we explain how the native C++ syntax is preserved in constraint formulas, such as in 2 <= z <= 5 && x + z <= 7. The logical connective && is supported by the C_LIN class. In turn, each of the C++ expressions 2 <= z <= 5 and x + z <= 7 must yield an object of type C_LIN. This is done by overloading operators <= and +. Clearly, in such an expression, x and z must be C++ variables that have already been declared within an appropriate C++ scope (the usual C++ scoping rules apply). The type for the C++ variables x and z is a special class, called CST_Var, which keeps inside the a constraint variable name, i.e. a string. It is convenient, although not required, to use the same name for a constraint variable name and the corresponding C++ variable into which the constraint variable is assigned. Each CST object keeps inside its free constraint variables that can be also shared among different CST objects.

8 Optimization by Approximation-based Filtering and Indexing

General optimization of object-oriented queries (e.g. ENCORE [Zdo89], O2 [ea90], POSTGRESS [SRH90]) and monoid comprehensions in particular, (e.g. [FM95]), as well as optimization in presence of expensive predicates [CS93, HS93] is outside the scope of this paper; We concentrate here on approximation-based filtering, regrouping and indexing [BW95], that EOSCUBE is designed to support. More specifically, we describe, mostly by examples, the EOSCUBE primitives for *approximation* and *inverse groupings* [BW95] and *indices* and special purpose algorithms.

To understand the idea, we use a modification of an example from [BW95] of the query: "find all trajectories passing over the Fairfax county". It will be assumed here that a set of 4D aircraft trajectories as well as a map is stored in the database. A trajectory is assumed to have a piece-wise linear representation, i.e. it is represented as a `DC_LIN` CST object

$$\bigvee_{i=1}^{n} (t_{i-1} \leq t < t_i \wedge x = a_{i,1}t + b_{i,1} \wedge y = a_{i,2}t + b_{i,2} \wedge z = a_{i,3}t + b_{i,3})$$

Where x, y, z are variables for a location, t is a time variable, and t_{i-1}, $a_{i,1}, b_{i,1}, a_{i,2}$, $b_{i,2}, a_{i,3}, b_{i,3}$, $1 \leq i \leq n$, are constants. Note that for each i, $(t_{i-1} \leq t < t_i \wedge x = a_{i,1}t + b_{i,1} \wedge y = a_{i,2}t + b_{i,2} \wedge z = a_{i,3}t + b_{i,3})\}$ describe the movement equations for the time interval $[t_{i-1}, t_i)$, for the constant 3-D velocity vector $(a_{i,1}, a_{i,2}, a_{i,3})$, starting from the point $(b_{i,1}, b_{i,2}, b_{i,3})$. `All_trajectories` is a variable of type `SET<DC_LIN*>`, i.e. a set of pointers to trajectories. The Fairfax county is assumed to be represented as a polygon,[9] i.e. as a `C_LIN` CST object `Fairfax_area` in variables x and y. The query can be directly expressed in EOSCUBE as

```
SELECT traj
INTO {Set<DC_LIN*>} result
FROM All_Trajectories AS {DC_LIN*} traj
WHERE  MUT_SAT(*traj, Fairfax_area )          // Note: MUT_SAT
                                              //   on DC_LIN
```

or, if `MUT_SAT` is expressed, in turn, as a monoid comprehension:

```
SELECT traj
INTO {Set<DEC_LIN*>} result
FROM All_Trajectories AS {DC_LIN*} traj
```

[9] in fact, it is not convex, but we'll assume that to simplify the example

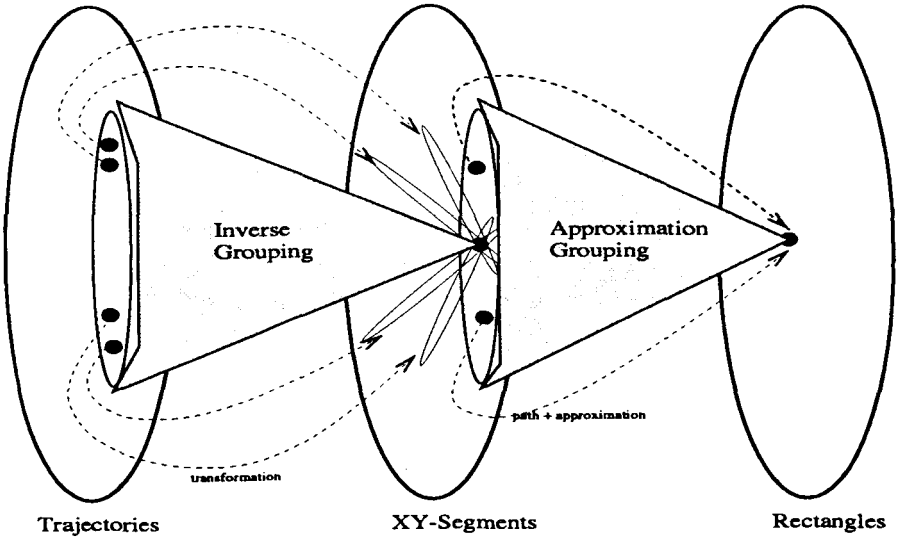

Figure 5: Inverse and Approximation Grouping

```
WHERE SELECT MUT_SAT(*segment,Fairfax_area)   // Note: MUT_SAT
      INTO {Some}                             //   on C_LIN
      FROM *traj AS {C_LIN*} segment          // SELECT returns
                                              //  True of False
```

which is an expensive query if evaluated directly.

Inverse Grouping(IG)

To optimize the last query, we can first use the *inverse grouping*, described graphically in Figure 7. Intuitively, each trajectory can be viewed as composed of 4D-segments, and each segment has a projection, say an `xy-segment` on the horizontal plane `x,y`.

Now, consider a collection of all `xy-segments` that originate from all the trajectories. The *inverse grouping primitive* tracks back, for each `xy-segment` in the collection, the set of all the trajectories from which this `xy-segment` came.

The correspondence between an `xy-segment` and the set of trajectories is captured through a parameterized C++ class `G_Pair<T1,T2>` which represents a pair containing a reference to an object of type T1 (`C_LIN*` in our case) as its first element and a reference to a set of objects of type T2 (`DC_LIN*` in our case) as its second element. The inverse grouping is captured through a special C++ class which essentially represents a set of `G_Pair`-s. To create an inverse grouping the user must create an instance of that class, passing the original collection as a constructor pa-

rameter. Assuming that All_Traj_IG has been created as the inverse grouping for the All_Traj collection, we can now re-write the previous query as follows:

```
SELECT traj
INTO {Set<DC_LIN*>} result
FROM  All_Traj_IG AS {G_Pair<C_LIN*,DC_LIN*>*} G_pair
//
DEFINE xy_segment AS {C_LIN*} G_pair->first
WHERE MUT_SAT( *xy_segment, Fairfax_area )        // Note: MUT_SAT
                                                  //   on C_LIN
DEFINE trajectories_of_xy_segment
       AS {Set<DC_LIN*>*} G_pair->second
FROM  *trajectories_of_xy_segment AS {DC_LIN*} traj
```

First note that the new query is equivalent to the previous one. The reason is that a trajectory passes over the Fairfax_area if and only if one of its xy-segment intersects the Fairfax_area. The new query, however, is likely to perform better. Note that different trajectories may "share" the same xy-segment, for example if one passes exactly above another. Therefore, the previous query will apply the MUT_SAT (i.e. intersection) test multiple times for every shared xy-segment. Whereas, in the last query we eliminate such duplication by checking MUT_SAT for each xy-segment only once and then quickly retrieving the corresponding trajectories using the inverse grouping.

Approximation Grouping(AG)

Even though we have minimized the number of the MUT_SAT checks, we know that MUT_SAT on C_LIN is relatively expensive. To optimize further, we can approximate each xy-segment with a minimum bounded box (MBOX), which is of type C_RANGE. Then, before testing MUT_SAT on C_LIN it can be tested first on MBOXes, which is cheaper. Also, it opens opportunities for indexing. This is the purpose of the *approximation grouping* primitive, which we introduce next. Intuitively, approximation grouping takes a collection of objects and creates another collection. The elements of the latter are approximations of the objects from the original one (see dashed lines in Figure 7). The approximation grouping, similar to the inverse one, must track back, for each approximation, the set of original objects that yield this approximation.

The implementation of the approximation grouping is similar to that of inverse grouping. It uses the same class G_Pair<T1,T2> which now represents the relationship between an approximation and the corresponding objects. The approximation grouping is constructed by creating an object of a special class, passing the original

collection and the approximation method as constructor parameters. As in the case of inverse grouping, the object being created represents a set of G_Pair-s. To continue the optimization of the last query the original collection in our case would be the inverse grouping collection All_Traj_IG. Recall that All_Traj_IG is a set of inverse grouping pairs G_Pair<C_LIN*,DC_LIN*>. If we define an approximation of an inverse grouping pair as an approximation of its first element (i.e. xy-segment) then we can create an approximation grouping object All_Traj_IG_AG which would be a set of G_Pair<C_RANGE*,G_Pair<C_LIN*,DC_LIN*>*>-s.

We can now re-write the last query as follows:

```
SELECT traj
INTO {Set<DC_LIN*>} result
//
FROM All_Traj_IG_AG AS {G\_Pair<C_RANGE*,G\_Pair<C\_LIN*,
                                          DC\_LIN*>*>} AG_pair
 DEFINE min_box_of_xy_segment AS {C_RANGE*} AG_pair->first
 WHERE  MUT_SAT(*min_box_of_xy_segment,  // On C_RANGE;
            min_box_of_Fairfax)          // precomputed outside
                                         // of the query
 DEFINE Candidate_Traj_IG AS {Set<G_Pair<C_LIN*,DC_LIN*>*>*}
                                                AG_pair->second
 //
 FROM  *Candidate_Traj_IG  AS {G_Pair<C_LIN*,DC_LIN*>*} IG_pair
 DEFINE xy_segment  AS {C_LIN*} IG_pair->first
 WHERE Mut_Sat( *xy_segment, Fairfax_area )       // Note: MUT_SAT
                                                  //   on C_LIN
 DEFINE trajectories_of_xy_segment
      AS {Set<DC_LIN*>*} IG_pair->second
 FROM  *trajectories_of_xy_segment AS traj
```

Indexed Approximation Grouping(IAG)

The last query involves the retrieval of rectangles that are mutually consistent with a given one. Another optimization possibility is to maintain an index on the collection of rectangles. This is done using the *indexed approximation grouping*, instead of the approximation grouping. The IAG has the same functionality as the approximation grouping with the following differences: 1) the first element of an IAG pair is always a rectangle (i.e. of type C_RANGE) and 2) there is a kD-tree index imposed on the rectangles. The IAG is constructed by creating an object of a special class which, in addition to the approximation grouping functionality, contains member functions for

search. In our example we create an indexed approximation grouping object named `All_Traj_IG_IAG` . As in the case of approximation grouping, `All_Traj_IG_IAG` is a set of `G_Pair<C_RANGE*,G_Pair<C_LIN*,DC_LIN*>*>`-s. In the query we invoke the `MUT_SAT(C_RANGE)` method on that object. The method returns, for each MBOX (of type `C_RANGE`), the set of all `G_Pair<C_LIN*,DC_LIN>*`-s for which the corresponding rectangles intersect that MBOX.

Note, that the returned set is not a physical set collection, but rather a structure allowing to iterate over its elements (and thus no intermediate evaluation is necessary when used within monoid comprehension). The last query can be re-written as follows:

```
SELECT traj
INTO {Set<DC_LIN*>} result
//
FROM All_Traj_IG_IAG.MUT_SAT(min_box_of_Fairfax)
  AS {Set<G_Pair<C_LIN*,DC_LIN*>*>*} Candidate_Traj_IG
//
FROM  *Candidate_Traj_IG  AS {G_Pair<C_LIN*,DC_LIN*>*} IG_pair
DEFINE xy_segment  AS {C_LIN*} IG_pair->first
WHERE Mut_Sat( *xy_segment, Fairfax )          // Note: MUT_SAT
                                               //  on C_LIN
//
DEFINE trajectories_of_xy_segment AS {Set<DC_LIN*>*} IG_pair->second
FROM  *trajectories_of_xy_segment AS traj
```

It is important to note, that, while we intuitively explained the use of the IG, AG and IAG by examples, these primitives can be applied to any `CollectionMonoid<A>`. For the IG the user needs to provide a transformation producing, for each element of type `A` an instance of (another) commutative and idempotent monoid (see [BW95] for details). For the AG and IAG, an approximation of elements of type `A` must be provided by the user. The IG, AG and IAG are used to facilitate the query transformation rules supporting approximation-based filtering (by using less expensive predicates first) and indexing (see [BW95]).

9 Related Work

No technology for declarative and efficient querying of databases involving constraint objects exists today. Applications of the kind discussed are typically implemented by special purpose programs; while these programs may use database and constraint programming tools, they typically require a considerable programming effort and

are not flexible to changes. In addition, they do not perform overall optimization that interleaves database, mathematical programming and computational geometry manipulation techniques. Existing DBMS do not manage constraints as persistently stored data [10]. Constraint Logic Programming [?, ?, ?], on the other hand, was not designed to deal with large amounts of persistent data. Extensions of DBMS with spatio-temporal operators [OM88, Gut89, Wol89, HC91] typically (1) are limited to low (two- or, at most three-) dimensional space, (2) have query languages restricted to predefined spatio-temporal operators, and (3) lack global economical filtering and deep optimization.

There has been work on the use of constraints in databases, earlier of which include [Klu88, ?, ?, ?, BS89]. The pioneering work [?] proposed a framework for integrating abstract constraints into database query languages by providing a number of design principles, and studied, mostly in terms of expressiveness and complexity, a number of specific instances. The work [HHLvEB89] considered polynomial equality constraints, adopting local propagation steps for reasoning on constraints. A restricted form of linear constraints, called *linear repeating points*, was used to model infinite sequences of time points [KSW90, ?, NS92]. More recent works on deductive databases [MFPR90, SR92, KS92, LS92] considered the manipulation and repositioning of constraints for optimizing recursion. Algorithms for constraint algebra operators such as constraint joins, and generic global optimization were studied in [BJM93], and constraint approximation-based optimization in [BW95]. The work [KRVV93] proposed an efficient data structure for secondary storage suitable for indexing constraints, that achieves not only the optimal space and time complexity as priority search trees [McC85], but also full clustering. The work [BLLM95] proposed an approach to achieve the optimal quality of constraint and spatial filtering. A number of works consider special constraint domains: integer order constraints [?]; set constraints [Rev95]; dense-order constraints [?]. Linear constraints over reals drew special attention [ABK95, ?, BJM93, BK95, BLLM95, ?, ?]. The use of constraints in spatial database queries was addressed in [?]. The work [SRR94] used constraints to describe incomplete information. Constraint aggregation was studied in [Kup93].

DISCO (Datalog with Integer and Set order COnstraints) is a constraint database system being developed at the university of Nebraska [BR95]. DISCO incorporates a highly expressive family of constraints. However, its query language has time complexity exponential in the size of a database; hence DISCO's applicability to real-size database problems is not clear. Further, DISCO does not support the standard database features such as persistent storage, transaction management and data integrity.

[10]Note, integrity constraints used in conventional databases are not data, but rather something the data must satisfy.

10 Lessons Learned

We first summarize some lessons we learned while building the EOSCUBE system.

The two-layer implementation of the constraint algebra was a simple and extensible solution to the problem of finding the appropriate C++ structures. The experiences learned while implementing the algebra can be applied to the C++ implementation of other similar multi-typed algebras.

Using ObjectStore and CPLEX as underlying components allowed us to concentrate on issues related to constraint implementation and to ignore many common aspects such as the simplex algorithm, persistence, data integrity etc.

However, in the process of development many problems came up while coding and maintaining some ObjectStore-specific parts. The goal was to put the library on a higher level, making it more flexible and uniform to use. Also, we did not want to tie the library design to the specific components used underneath. Rather, we aimed to make it portable enough to be easily transfered on the top of another object manager and/or LP package. This has been achieved by implementing an intermediate interface between EOSCUBE and the underlying components, which isolated the developers and users of the system from ObjectStore technicalities. In order to use a different object manager/LP package, only a relatively small part of the code will have to be re-written. A lot of effort has gone in making this interface both portable and easy to use.

Still, a fair amount of time has been spent on the implementation of the supporting structures such as search trees and sparse matrices. Even though there are some C++ packages that handle these issues, none of them was flexible enough to be directly plugged in EOSCUBE.

Currently EOSCUBE does not parse the full C++ grammar. Instead, there is a pre-compiler that passes its output over to the C++ compiler. EOSCUBE queries are embedded in hosting C++ programs and are allowed to use variables declared in the appropriate scope (with some restrictions). During the implementation of this scheme we encountered some non-trivial technical problems that required much effort to resolve.

It is important to note that C++ was the best pragmatic choice for our purposes. Its compactness and expressiveness power enabled us to make things that would be hardly possible in any other programming language. The library extensively uses templates, operator overloading and multiple inheritance. Those features also provided higher reusability of components which allowed us to reduce the total amount of the source code (around 11,000 lines now, not including the commercial components).

Even though the query language we presented is an optimization-level language in which evaluation plans can be explicitly expressed, we found that the optimization primitives are not currently automated to the extent they can possibly be. For

example, similar to a regular index, the grouping primitives require dynamic maintenance. If there is an update in the original collection, the corresponding change must be made in the grouping structures. EOSCUBE does not support the dynamic updates in the current implementation. The work in this direction is currently being performed.

We have described the work on the development of the first constraint object-oriented database system. Our work aims at the developing a practical and useful technology for a wide variety of important application realms, for which no existing technology is applicable. For example, EOSCUBE can be directly used to implement the real-life data fusion and sensor management system for air-space command and control [ABK95]. EOSCUBE is a deeply optimizable and extensible system, striking the balance between expressiveness and computational complexity.

Many research questions remain open (see [Bro] for an overview): in constraint modeling and canonical forms, data models and query languages, indexing and approximation-based filtering, and, most importantly, special constraint algebra algorithms for specific domains and global optimization.

Part V
Global Optimization using Workflows: Work in Progress

11 Workflow Systems

A workflow is a collection of cooperating, coordinated activities designed to carry out a well-defined complex process. In the context of EOSCUBE, the activities considered are computation of interrelated EOSDIS and other scientific products.

A number of workflow representation frameworks has been proposed, the most common being *control flow graphs*, *triggers* (also known as event-condition-action rules) and temporal constraints.

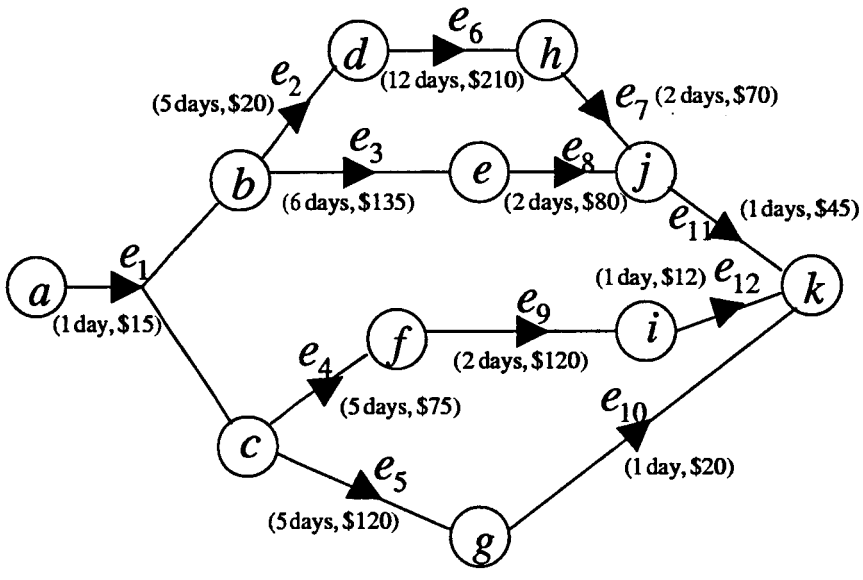

Figure 6: Control Flow Graphs

To exemplify a control-flow graph, consider an abstract example depicted in the graph in Figure 1. The example depicts achieving a global task (e.g., generating a

set EOSDIS products for the sattellite data of 24 hours) using activities E1 through E12 (e.g., involving generating various levels of EOSDIS products, and possibly some auxiliary intermediate results) going through the starting state A to the final state K.

The graph indicates that activity E1 must precede all other activities, leading to reaching states B and C, both of which must be pursued to reach the final state K (the completion of the house construction). There are two alternatives to reach state J from state B: through D and H or through E. The first possibility must involve activities E2, E6, and E7, in this order; the second possibility must involve activity E8 which must be preceded by the activity E3. Similarly, there are two ways to reach node K from node C.

Today, EOSDIS products do not have this choice of more than one way of computing. However, as many variants for spatial correlation exemplify, in order to compute a series of products, a number of generation programs can be implemented (especially if it only requires little effort). For example, instead of directly computing a product from its inputs (lower-level products), those inputs can be preprocessed into a database that will have auxiliary data structures and features (e.g., indexing and clustering) and then more efficient evaluation strategies can be chosen. Such strategy may well pay off if a many products are used the preprocessed data, thus amortizing the preprocessing effort. Or, one can consider a strategy when a number of products are evaluated "on the fly" in parallel, when input streams are used in more then one computation. That is, when a block is retrieved from secondary or tertiary storage, or produced by a product program, it is used in as many places as possible, while it still reside in main memory. Advantages of each choice depend on the overall data and system state. Workflow optimization is a natural way to make optimal choices dynamically depending on the system state.

The control flow graph is most appropriate for depicting the local execution dependencies of the activities in a workflow; it is a good way to visualize the overflow of control. Control flow graphs are the primary specification means in most commercial implementations of workflow management systems. As seen in the above example, a typical graph specifies the initial and the final activity (or a state) in a workflow, the successor activities for each activity in the graph, and whether these successors must all be executed concurrently, or it suffices to execute just one branch non-deterministically. Intuitively, concurrent execution corresponds to AND edges in the graph, whereas non-deterministic choice corresponds to OR edges. Edges in a control flow graph can be labeled with *transition conditions*. The condition applies to the current state of the workflow (which, in a broad sense, may include the current state of the underlying database, the output of the completed tasks, the current time, etc.). When the task at the tail of an edge completes, the task at the head can begin only if the corresponding transition condition evaluates to true. The Workflow Management

Coalition [10] identifies additional controls, such as loops and sub-workflows.

In the following, we present some work in progress on workflow optimization, to be used for global optimization in multi-product generation environment. This is a joint work with Larry Kerschberg from GMU and Samuel Varas from University Chile at Santiago, and currently visiting GMU.

12 Workflow Representation and Scheduling

In this section we introduce a workflow graph representation called Workflow Graphs ($\mathcal{G}$), and we show how the workflow characteristics are represented in such graphs. Intuitively, Figure 1 represents a Workflow Graph. The notion of Workflow Graph is formalized as follows.

Definition 1. *A* Workflow Graph $\mathcal{G}$ *is a triple* (V, E, W) [11], *where:*

1. $V = \{v_1, \ldots, v_n\}$ *is the set of* nodes *(called events),*
2. $E = \{e_1, \ldots, e_m\}$ *is a set of* edges *(called activities). Each* e_i *in* E *is a pair* $(T(e_i), H(e_i))$ *where* $T(e_i) \in V$, *called the* tail *of* e_i, *and* $H(e_i) \subseteq V - \{T(e_i)\}$ *is called the* head *of* e_i, *and*
3. $W : E \rightarrow \mathbb{R}^k$ *is a weight function that maps each activity* e_i *in* E *to a* k*-tuple of weights* $(w_1^i, \ldots, w_k^i)$.

Note that the *tail* of e_i ($T(e_i)$) represents the incoming node of e_i, and *head* of e_i ($H(e_i)$) represents the set of all outcoming nodes of e_i.

The Workflow Graph that corresponds to Figure 1 is as follows. N = $\{a, b, c, d, e, f, g, h, i, j, k$ E = $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}\}$, and e_1 is $(a, \{b, c\})$, e_2 is $(b, \{d\})$, e_3 is $(b, \{e\})$, e_4 is $(c, \{f\})$, e_5 is $(c, \{g\})$, e_6 is $(d, \{h\})$, e_7 is $(h, \{j\})$, e_8 is $(e, \{j\})$, e_9 is $(f, \{i\})$, e_{10} is $(g, \{k\})$, e_{11} is $(j, \{k\})$, and e_{12} is $(i, \{k\})$.

We denote by $O(v_i)$ the set of all outcoming edges of node v_i, i.e., $O(v_i) = \{e_j \mid T(e_j) = v_i\}$, and by $I(v_i)$ the set of all incoming edges at node v_i, i.e., $I(v_i) = \{e_j \mid v_i \in H(e_j)\}$. In addition, we denote by $\mid e_i \mid$ the *cardinality* of e_i, i.e., the total number of incoming and outcoming nodes of e_i. The size of a Workflow Graph $\mathcal{G}$, denoted by $size(\mathcal{G})$, is defined as $\sum_{e \in E} \mid e_i \mid$.

Now, we define workflow graph paths and workflow execution plans on Workflow Graphs.

Definition 2. *Let* (V, E, W) *be a Workflow Graph. Then, a* path *from a node* s *to a node* t *of length* q, *denoted by* $P_{st}(q)$, , *is a sequence of nodes and edges,* (s =

[11] Workflow Graphs are an extension of directed *F-hypergraphs* [?].

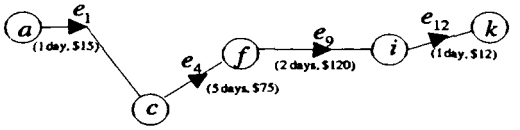

Figure 7: Path on Workflow Graph

$v_1, e_1, v_2, e_2, \ldots, v_j, e_j, v_{j+1}, \ldots, v_q, e_q, v_{q+1} = t)$, *such that for all* j, $1 \leq j \leq q$, $v_j = T(e_j)$ *and* $v_{j+1} \in H(e_j)$.

As example, in Figure 2 $(a, e_1, c, e_4, f, e_9, i, e_{12}, k)$ is a path from node a to node k.

If $t = s$, $P_{st}(q)$ is said to be a *cycle*. In a *simple* path all edges are distinct, and a simple path is *elementary* if all nodes $v_1, v_2, \ldots, v_{q+1}$ are distinct. *Simple* and *elementary cycles* we define similarly. A path is said to be *cycle-free* if it does not contain any subpath which is a cycle.

To execute a workflow we need to perform a set of activities with a predefine order. We introduce the concept of *execution plan* to formalize how workflow can be executed.

To understand intuitively, an example of execution plan is depicted in Figure 3, where workflow starts at event a, then activity e_1 is executed. After that, activities e_2, e_6, e_7, and e_{11} are executed sequentially. In parallel to that, activities e_5, e_9, and e_{12} are executed sequentially. More formally.

Definition 3. *Let* (V, E, W) *be a Workflow Graph. An* hyperpath, *also called execution plan interchangibly, from an event* s *to an event* t, *denoted by* Π_{st}, *is a minimal Workflow Graph* (V_Π, E_Π, W_Π), *such that:*

1. $E_\Pi \subseteq E$
2. $s, t \in V_\Pi \subseteq V$
3. *For every* $v \in V_\Pi$, v *is connected to* t *in* (V_Π, E_Π, W_Π) *by a cyclic-free simple path, and*

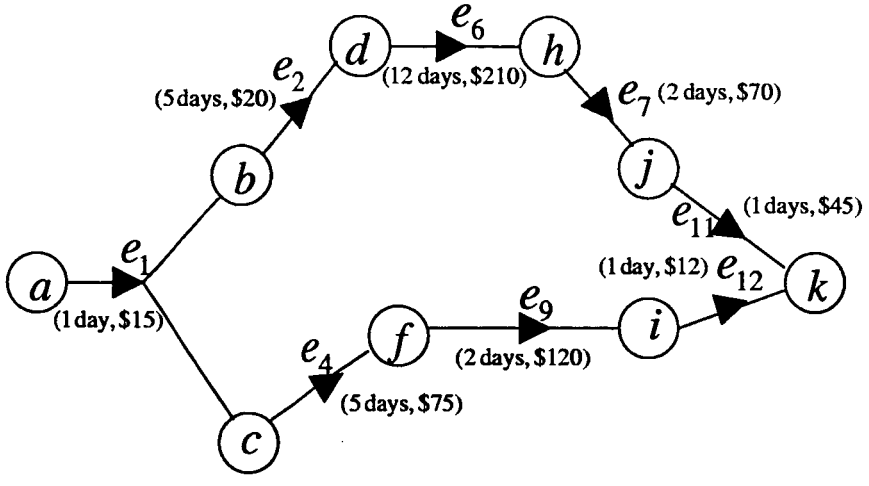

Figure 8: Execution Plan on Workflow Graph

4. W_Π *and* W *agree on* E_Π, *i.e., for every* $e \in E_\Pi$, $W_\Pi(e) = W(e)$.

Figure 3 depicts an example of an execution plan from event a to event k. In this case $V_\Pi = \{a, b, c, d, f, h, i, j, k\}$, $E_\Pi = \{e_1, e_2, e_4, e_6, e_7, e_9, e_{11}, e_{12}\}$, and $W_\Pi = \{W_1, W_2, W_4, W_6, W_7, W_9, W_{11}, W_{12}\}$.

Note that there are 4 possible workflow execution plans in the Workflow Graph depicted in Figure 1. In general, for larger or more complex Workflow Graphs the number of possible execution plans grows exponentially. Therefore, analyzing execution plans' properties on a Workflow Graph could be a hard problem. We consider the following problem characterization to analyze workflow execution plans' properties.

Let $\mathcal{G} = (V, E, W)$ be a Workflow Graph, and Π the set of all execution plans from a node v_o to a node v_d on $\mathcal{G}$. We suggest associating an optimal criterion f to execution plans on Π, i.e., $f : \Pi \to \mathbf{R}$. Also, execution plans in Π may be constrained, i.e., constraints $\mathcal{C}(\rho_k)$ are given, $\rho_k \in \Pi$. Then, we consider the following problems.

1. Consistency: Determine if there exists an execution plan that satisfies $\mathcal{C}$.

2. Verification:

3. Scheduling: Find an execution plan where $\mathcal{C}$ holds.

4. Scheduling Optimization: Find the *best* workflow execution plan between a node v_o and a node v_d, i.e., the optimization problem is as follows:

$$\begin{aligned} &\min_{\rho \in \Pi} \{f(\rho_k)\} \\ &s.t.\, \mathcal{C}(\rho_k) \end{aligned} \tag{1}$$

In general, complexity of (1) corresponds to the number of elements in set Π, which are in the worse case exponential in the size of $\mathcal{G}$. A possible solution strategy to (1) corresponds to to examines all their elements. However, there are alternative mechanisms to solve (1) rather than an exhaustive enumeration. Recursive optimization formulations have been proposed, but in general, the problem is a NP-hard [?, ?].

In the following sections we analyze different cases of problem (1), and provide algorithms to solve effectively.

13 Unconstrained Localizable Scheduling

In this section we consider the case of finding a solution of (1) when there are no constraints other than those defined by the graph. Still the problem is NP-hard in general, and we first consider a special important class of objective functions, that we call *locally computable.*

13.1 Problem Statement

Definition 4. *Let $\mathcal{G} = (V, E, W)$ be a Workflow Graph, and Π be the set of all hyperpaths in $\mathcal{G}$. We say that a function $f : \Pi \to \mathbb{R}$ is a* locally computable *function if for all pair of nodes (v_o, v_d), such that $\rho_{od} \in \Pi$, f can be expressed as follows:*

$$f(\rho_{od}) = \begin{cases} f_0, & \text{if } v_o = v_d \\ w(e) + \mathcal{F}(\{f(\rho_{kd}) \mid v_k \in H(e)\}), & \text{if } v_o \neq v_d. \end{cases}$$

where ρ_{od} starts with edge e, $\mathcal{F}$ is a nondecreasing function in terms of $\{f(\rho_{kd}) \mid v_k \in H(e)\}$, and f_0 is a real value.

An important *locally computable* function corresponds to the workflow execution time, i.e., the necessary time to carry out the workflow. In this case, w represent the activity execution time for each $e_i \in E$, and function $\mathcal{F}$ is as follows.

$$\mathcal{F}(\{f(\rho_{kd}) \mid v_k \in H(e)\})\} = \max_{v \in H(e)} \{f(\rho_{kd})\} \tag{2}$$

Next proposition provides a mechanism to re-write (1) as a recursive optimization problem.

Proposition 1. *Let $\mathcal{G} = (V, E, W)$ be a Workflow Graph, Π_{od} be the set of all hyperpaths from v_o to v_d, and $f : \Pi \rightarrow \mathbb{R}$ be a locally computable function. Then,*

$$\begin{aligned} &1.\ \min_{\rho \in \Pi} f(\rho) = f_0 \\ &2.\ \min_{\rho \in \Pi} f(\rho) = \min_{e \in O(v)} \{w(e) + \mathcal{F}(\{\min_{\rho' \in \Pi} f(\rho') \mid v_k \in H(e)\})\} \end{aligned} \tag{3}$$

Proof. □

Problem (3) correspond to the *generalized Bellman's equations* [?, ?], and its solution is a generalization of the well-known *shortest-path* problem. Now, we present an algorithm to solve (3) in linear time.

13.2 Shortest Execution Plan Algorithm

The shortest execution plan algorithm [?, ?] assumes that $\mathcal{G}$ is ordered in inverse topological order, i.e., nodes are enumerated, such that the following condition is satisfied:

$$(T(e_k) = v_i) \wedge (v_j \in H(e_k)) \Rightarrow v_j < v_i. \tag{4}$$

The following algorithm $Acyclic - ITO$ [?, ?] has as input a Workflow Graph ($\mathcal{G}$), and its outputs are whether $\mathcal{G}$ is acyclic, and if it so, the inverse topological order. The algorithm is as follows [12].

Procedure Acyclic-ITO($\mathcal{G}$)
  for each $i \in V$ **do** $r_i = 0$
  for each $e = (\{i\}, H(e)) \in E$ **do** $r_i = r_i +$ | $T(e)$ |
  k = 0; Q = $\emptyset$
  for each $i \in V$ **if** $r_i = 0$ **then** Q = Q $\cup$ $\{i\}$
  while $Q \neq \emptyset$ **do**
    select and remove $u \in Q$
    k =k +1; $\pi_u = k$
    for each $e \in (\{i\}, H(e)) \in I(u)$ **do**
      $r_i = r_i - 1$
      if $r_i = 0$ **then** $Q = Q \cup \{i\}$
  if ($k = n$) **then**
    return "$\mathcal{WFG}$ is acyclic"
  else return "$\mathcal{WFG}$ is not acyclic"

[12] Q is implemented as queue

When $\mathcal{G}$ is acyclic, then π_u, for all $u \in E$, provides the inverse topological order. Since each edge is examine only one, complexity of $Acyclic - ITO$ correspond to $O(size(\mathcal{G}))$ [?].

Now, the algorithm to find the shortest execution plan in a Workflow Graph ($\mathcal{G}$). The input are the destination node v_d and the Workflow Graph $\mathcal{G}$. The algorithm is as follows.

Procedure SEP-Acyclic$(v_d, \mathcal{G})$
  for each $i \in V$ **do**
    $P_i = 0$
    if ($i = v_d$) **then** $f(i) = 0$ **else** $f(i) = \infty$
  for each $e_j \in E$ **do** $k_j = 0$
  for i = 1 **to** $|V|$ -1 **do**
    for each $e_j = (y, H(e_j)) \in I(i)$ **do**
      $k_j = k_j + 1$
      if $k_j = |H(e_j)|$ **then**
        $\eta = w(e_j) + \mathcal{F}(\{f(i) \mid i \in H(e_j)\})$
        if $(f(y) > \eta)$ **then**
          $f(y) = \eta$
          $P_y = e_j$

The output is a set $\{P_i \mid i \in V\}$ indicating, for each node i, which edge has been selected . Therefore, the execution path is constructed by a forward recursively enumeration, starting from the origin node of $\mathcal{G}$.

To analyze the complexity of $SEP - Acyclic()$ algorithm, we first note that each node and each edge is selected at most once. Therefore, the overall complexity is $O(size(\mathcal{G}))$ [?].

14 Constrained Localizable Scheduling

In this section we analyze the constrained problem (1) under *locally computable* objective functions assumption. In general, this is a harder problem than the unconstraint one, because feasibility (constraint satisfaction) needs to be checked at each hyperpath. We will consider a special case of constraints called *locally computable constraints*, which are defined as follows.

14.1 Problem Statement

Definition 5. *Let $\mathcal{G} = (V, E, W)$ be a Workflow Graph, and Π be the set of all hyperpaths to a node v_d in $\mathcal{G}$. Let $g : \Pi \to \mathbb{R}^+$ be a function and R be a positive real value. Then, we say that a constraint $s(\Pi_k) = R - g(\Pi_k) \geq 0$, $\Pi_k \in \Pi$, is a* locally computable constraint *if for all $\Pi_{od} = (V_\Pi, E_\Pi, W_\Pi) \in \Pi$, s can be expressed as follows:*

$$\begin{aligned} &s(\Pi_{dd}) = R \\ &s(\Pi_{od}) = -w_2(e_j) + \mathcal{S}(\{s(\Pi_{kd}) \mid v_k \in H(e_j)\}) \geq 0,\ \{e_j\} = E_\Pi \cap O(v_o) \end{aligned} \tag{5}$$

where $\mathcal{S}$ is a nondecreasing function in terms of $\{s(\Pi_{kd}) \mid v_k \in H(e_j)\}$. Function s is called slack constraint, *and $s(\Pi_{od})$ is the slack available at node v_o.*

This type of constraints can be associated with execution time (where R represents the maximum execution time required). Other characterization correspond to traversal cost, rank, or distance [?, ?].

Finding a feasible hyperpath recursively requires to define some criteria to discriminate at each step whether an edge may belong to a feasible hyperpath. We suggest as a criterion the *minimum feasible slack* defined as follows.

Definition 6. *Let $\mathcal{G} = (V, E, W)$ be a Workflow Graph, s be a slack constraint, and Π be the set of all possible hyperpaths from a node v_o to a node v_d. We say that $(r_1, r_2, \ldots, r_n)$ is a n-tuple of* minimum feasible slack *at nodes in V, defined by*

$$r_i = \begin{cases} \min_{\Pi\ \in \Pi}\{s(\Pi_{id}) \mid s(\Pi_k) \geq 0\}, & \text{if } v_i \in \bigcup_{\Pi\ \in \Pi} V_\Pi \\ 0 & \text{otherwise.} \end{cases}$$

Now we can formulate (1) as a constrained recursive optimization problem as follows.

Proposition 2. *Let $\mathcal{G} = (V, E, W)$ be a Workflow Graph, with $W = (w_1, w_2)$, and Π be the set of all hyperpaths from a node v_i to a node v_d. Let f be a locally computable objective function, s be a slack constraint, and $(r_1, \ldots, r_n)$ be a vector of minimum feasible slacks. Then, the optimal feasible hyperpath* [13] *from a node v_o to a node v_d problem (1) is equivalent to the following optimization problem.*

$$\begin{aligned} f(\Pi_{id}) = \min_{e\ \in O(v\ )} &\{w_1(e_j) + \mathcal{F}(\{f(\Pi_{kd}) \mid v_k \in H(e_j)\}) \mid \\ &s(\Pi_{id}) = -w_2(e_j) + \mathcal{S}(\{s(\Pi_{kd}) \mid v_k \in H(e_j)\}) \geq r_i\}, \\ &\forall\, v_i \in V - \{v_D\} \\ s(\Pi_{dd}) = R,& \\ f(\Pi_{dd}) = f_0.& \end{aligned} \tag{6}$$

[13] We assume that w_1 is related to the objective function and w_2 to the constraint

where $\mathcal{F}$ and $\mathcal{S}$ are nondecreasing functions, and f_0 is a constant and R is a positive constant.

Proof. □

14.2 Constrained Execution Plan Algorithm

The algorithm also requires a topological order of nodes, which is provide by algorithm $Acyclic - ITO$. Then, the set of all minimum feasible slack is calculated with the following algorithm.

Procedure MinFeSlack($\mathcal{G}$)
  for each $i \in V$ **do**
    if ($i = v_o$) **then** $r_i = 0$ **else** $r_i = \infty$
  for i = | V | **to** 1 **do**
    for each $k \in H(e_j) \mid i = T(e_j)$ **do**
      if $r_i + w(e_j) < r_k$ **then**
        $r_k = r_i + w(e_j)$

This algorithm examines all nodes and edges once, then, its complexity is $O(size(\mathcal{G}))$.

Finally, a modification of $SEP - Acyclic$ algorithm provides the solution of our constraint problem (6).

Procedure SCP-Acyclic$(v_d, \mathcal{G})$
  for each $i \in V$ **do**
    $P_i = 0$
    if ($i = v_d$) **then**
      $f(i) = 0$, $s(i) = R$
    else
      $f(i) = \infty$, $s(i) = 0$
  for each $e_j \in E$ **do** $k_j = 0$
  for i = 1 **to** $\mid V \mid$ -1 **do**
    for each $e_j = (y, H(e_j)) \in I(i)$ **do**
      $k_j = k_j + 1$
      if $k_j = \mid H(e_j) \mid$ **then**
        $\eta = w_1(e_j) + \mathcal{F}(\{f(i) \mid i \in H(e_j)\})$
        $\nu = -w_2(e_j) + \mathcal{S}(\{s(i) \mid i \in H(e_j)\})$
        if $(f(y) > \eta$ and $\nu \geq r_i)$ **then**
          $f(y) = \eta$
          $s(y) = \nu$
          $P_y = e_j$

Algorithm $SCP-Acyclic$ for each node analyzes if there going to be a enough slack to reach the root node. When there is no enough slack, this is an infeasible possibility, and then, it is not consider. Since this is just a variation of $SEP - Acyclic$, with additional evaluation (slack at each node), its complexity is $O(size(\mathcal{G}))$.

15 Additive Unconstrained Optimization

In this section we extend the concept of locally computable function an optimization. We reduce problem (1) under certain condition to a network flow (integer linear) problem, where the solution is always integer.

15.1 Problem Statement

Since locally computable functions require that their values being calculated just in terms of local elements, many types of functions cannot satisfy this condition. For example, total cost, total execution time, or any function where its value depends of an hyperpath. To overcome this limitation, we introduce *additive* functions defined as follows.

Definition 7. *Let $\mathcal{G} = (V, E, W)$ be a Workflow Graph, and Π be the set of all hyperpaths in $\mathcal{G}$ from a node v_o to the node v_d. Then, we say that a function $f : \Pi \to \mathbb{R}$ is a* additive *function if for all $\Pi_{od} = (V_\Pi, E_\Pi, W_\Pi) \in \Pi$, f can be expressed as follows:*

$$f(\rho_{od}) = \sum_{e \subset \rho} w(e) \tag{7}$$

Now, we introduce the concept of *hyperflow* on a Workflow Graph [?]. Intuitively, an hyperflow is a flow in hypergraphs, where for each node a conservation flow equation is specified. In particular, we consider that an unit flow is introduced in node v_o of $\mathcal{G}$ and a unit flow is gotten in node v_d. We define the following rule when an edge e has $\mid H(e) \mid \geq 2$: *for each unit incoming flow at edge e the outcoming flow is $\mid H(e) \mid$, i.e., additional flows are created for each $v_i \in H(e)$.*

Although previous rule preserves the unitary of the flow, it creates an additional problem by the artificial flow creation. To fix that, we introduce the concept of *h-artificial* flow edge at each node as follows.

Definition 8. *Let $\mathcal{G} = (V, E, W)$ be a Workflow Graph. We say that edge e_i is an* h_i-artificial *flow edge at node v_k if: (a) $\mid H(e_i) \mid \geq h_i$ and $h_i \geq 2$, and (b) there exists h_i different paths from e_i to v_k without common elements except e_i and v_k.*

From Figure 1, edge e_1 is a *2-artificial* edge to node k, i.e., a unit flow incoming to edge e_1 becomes in two units of flows incoming to node k.

Now, we can define formally the concept of unit hyperflow in a Workflow Graph as follows.

Definition 9. *Let $\mathcal{G} = (V, E, W)$ be a Workflow Graph, and $\theta(v_k)$ be a set of* h_i-artificial *flow edges at node v_k, $v_k \in V$. We say that an* unit hyperflow *from v_o to v_d in $\mathcal{G}$ is a function $x : E \to \{0\} \cup \mathbb{R}^+$ that satisfies the following conservation constraints:*

$$\sum_{e \in O(v)} x_j - \sum_{e \in I(v)} x_k - \sum_{e \in \theta(v)} (h_u - 1)\, x_u = \begin{cases} 1, & \text{if } v_i = v_o \\ 0, & \text{if } v_i \neq v_o \text{ and } v_i \neq v_d \\ -1, & \text{if } v_i = v_d \end{cases} \tag{8}$$

Note that (8) can be represented in a matricial way $A\vec{x} = \vec{d}$, where $\vec{x} = (x_1, \ldots, x_m)^t$, $\vec{d} = (1, 0, \ldots, 0, -1)^t$ [14], and a $n \times m$ matrix A, called *unitary incidence matrix*, where

[14] We consider that v_o and v_d are in the first and last position of $\vec{d}$ respectively.

its (i, j) element correspond to:

$$a_{ij} = \begin{cases} +1, & \text{if } v_i = T(e_j) \\ -1, & \text{if } v_i \in H(e_j) \\ 1 - h_u, & \text{if } e_u \in \theta(v_i) \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

Matrix A has two important properties, namely: (a) it is an integer matrix, i.e., all elements are integer, and (b) $\sum_{i=1}^{n} a_{ij} = 0$. Last property comes from the fact that for each edge e all incoming flow is balanced with the outcoming flow. Next proposition provides the basis for the solution of system $A\vec{x} = \vec{d}$.

Theorem 1. *Let $\mathcal{G} = (V, E, W)$ be a Workflow Graph, and $A\vec{x} = \vec{d}$ be the hyperflow constraint conservation on $\mathcal{G}$. If $A = (B\ R)$ and $\vec{x} = (\vec{x}_B, \vec{x}_R)$, where B is a base of $A\vec{x} = \vec{d}$. Then, for every base B of $A\vec{x} = \vec{d}$ there exists an hyperpath Π in $\mathcal{G}$ and vice versa.*

Proof. □

Now, we re-write the optimization problem (1) for additive functions as an hyperflow conservation problem, where a unit flow is sent from node v_o to a node v_d.

Proposition 3. *Let $\mathcal{G} = (V, E, W)$ be a Workflow Graph, Π be the set of all hyperpaths, $f : \Pi \to \mathbb{R}$ be an additive function, and $A\vec{x} = \vec{d}$ be the unit hyperflow constraint conservation of $\mathcal{G}$. Then,*

$$\begin{aligned} & min \sum_{e \in E} w(e_j)\, x_j \\ & s.t.\ A\vec{x} = \vec{d} \end{aligned} \tag{10}$$

Proof. □

Therefore, we can use the Simplex algorithm to solve (10), and the solution is an hyperpath that minimizes the objective function. Next section presents an algorithm to determine the *unitary incidence matrix* A.

15.2 Algorithm

The only remaining part from previous section is how matrix A is determined. The algorithm consists in two steps, where the first one determines an intermediate matrix

A' as follows.

$$a'_{ij} = \begin{cases} +1, & \text{if } v_i = T(e_j) \\ -1, & \text{if } v_i \in H(e_j) \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

Then, this intermediate matrix A' is fixed by adding the $h_i - artificial$ edges in its values. This step assume that the Workflow Graph is in inverse topological order, and each edge e has ψ a set of pairs (e, K) to indicate the k^{th} node in the $H(e)$. This step is as follows.

Procedure IncidenceMatrix$(A', \mathcal{G})$
  for each $v_i \in V$ **do** $k_i = 0$
  for each $e_j \in E$ **do**
    $\psi_j = \emptyset$
    for $q = 1$ **to** $\mid H(e_j) \mid$ **do**
      $\psi_j = \psi_j \cup (j, q)$
  for i $= \mid V \mid$ **to** 1 **do**
    for each $e_j \in \Omega(i)$ **do**
      if $\mid \alpha_j \mid \geq 2$ **then**
        $a'_{ij} = a'_{ij} - \mid \alpha_j \mid$
        $\psi_j = \psi_j - \alpha_j$
      if $\mid \psi_j \mid \geq 2$ **then**
        for each $e_k \in O(i)$ **do**
          $\psi_k = \psi_k \cup \alpha_j$

Where set $\Omega(i) = \{e \mid \exists (e, r) \in \cup_{u \in I(v)} \psi_u\}$ and set $\alpha_j = \{(e, q) \mid \exists (e^*, r) \in \cup_{u \in I(v)} \psi_u, e = e^* \wedge r \neq q\}$.

Algorithm $IncidenceMatrix(A', \mathcal{G})$ analyzes each node once, and for each node calculate set $\Omega(i)$ and for element in such set calculates α_j. Then, the overall complexity corresponds to the complexity of set $\Omega(i)$ times complexity of set α_j times the number of nodes in $\mathcal{G}$, i.e., $O(\mid V \mid \times O(\Omega) \times O(\alpha))$.

16 Extended Workflow Scheduling Problem

In this section we extend the workflow scheduling problem considering that there are some constraints over a subset of activities indicating if those activities may or may not mandatory be executed. As example of this type of constraints consider the case

when two activities e_1 and e_2 are simultaneously allowed or disallowed to appear in an feasible execution plan, i.e., the logic constraint is $(\neg e_1 \wedge \neg e_2) \vee (e_1 \wedge e_2)$.

16.1 Problem Statement

In general, finding a feasible execution plan under such constraints makes the problem NP-hard, because there is necessary to check all combinations of allowed activities. Before we formulate our optimization model we need to define the notion of *feasible* instantiation as follows.

Definition 10. *Let $\mathcal{G} = (V, E, W)$ be a Workflow Graph, $\hat{\mathcal{C}}$ be a constraint over activities $e_1, \ldots, e_k$, and $\vec{\omega} = (\omega_1, \ldots, \omega_k)$ be a Boolean vector, such that $\omega_i = TRUE$, $1 \leq i \leq k$, if e_i must be in the execution plan, $FALSE$ if e_i must not be in the execution plan. Then, we say that an instantiation of $\vec{w}$ is* feasible *if it satisfies $\hat{\mathcal{C}}$.*

Note that there are 2^k possible instantiations of $\vec{\omega}$ to check whether is feasible or not, which is the source of the NP-hardness.

The optimization problem can be formulated similar to (1) with the additional constraint $\hat{\mathcal{C}}(e_1, \ldots, e_k)$. Then finding the best execution plan between node a v_o and a node v_d can be formulated as

$$\begin{aligned} &\min_{\Pi \in \Pi} \{f(\Pi_k)\} \\ &s.t.\, \mathcal{C}(\Pi_k) \\ &\quad \hat{\mathcal{C}}(e_1, \ldots, e_k) \end{aligned} \tag{12}$$

In general, solving problem (12) requires exponential time. However, when a feasible instantiation of $\vec{\omega}$ is given, problem (12) is similar to (1). This observation is used to propose a local search algorithm [?, ?, ?] in the next section. In the following, we consider that function f and constraint $\mathcal{C}$ satisfy the condition from sections 13, 14, and 15.

16.2 Local Search Algorithm

A local search algorithm is structured as follows: *a number of local searches are performed, where for each one, the algorithm checks if the local optimum is better than the current objective function value.* This procedure is repeated until there is no acceptable neighborhood possible or some criteria are satisfied (number of iterations, minimum value, etc.).

In this case acceptable neighborhoods correspond to feasible instantiations of $\vec{\omega}$. Therefore, we need a procedure to generate those neighborhoods. There are many

possible ways to implement this procedure, (sequential, random ,etc.). However, we do not discuss them here, and we just consider that there is such a procedure called $GetNextNeighborhood()$.

The local search framework get a $\vec{\omega}$ instantiation, let say $\vec{\omega}_k$, and modifies $\mathcal{G}$ with that information (assigns an huge cost to the activities with FALSE). The resulting problem is one of the problem formulated in sections 13, 14, and 15. Therefore, we can use the their algorithms to solve the local search (Step 1). The local search framework is as follows.

Step 0. Assign 0 to k, $\vec{\omega}_k = GetNextNeighborhood()$, and $f^* = \infty$.

Step 1. Assign $w_i = \infty$ if $\omega_i = FALSE$, and perform a local search, i.e., solve the shortest execution plan, getting solution Π_k.

Step 2. if $f(\Pi_k) < f^*$, then $f^* = f(\Pi_k)$, and $\Pi^* = \Pi_k$.

Step 3. Increase k by 1, select a new $\vec{\omega}$ instantiation, and go to step 2. If there is no $\vec{\omega}$ instantiation, go to step 4.

Step 4. Report objective function f^* and solution Π^*.

Local search framework complexity, in the worst case, is exponential in terms of k, i.e., the number of activities involved in constraint $\hat{C}$. In particular, complexity is $O(2^k \times \mid E \mid \times size(\mathcal{G}))$, since we have to execute local search algorithm (shortest execution path) at most 2^k times. However, using some additional stop criteria, it is possible get *good* solutions without examine all possible combinations.

Part VI
Conclusions and Suggested Future Directions

Main Conclusions in Phase 1:

- EOSCUBE has the potential for significant productivity gain in specification and generation of EOSDIS and other scientific products
- Generation of scientific products from real data sets is feasible using the EOSCUBE prototype
- An industrial-strength EOSCUBE implementation will be necessary for deployment and massive use of the system.
- The EOSCUBE language should allow incremental extensions, which are unavoidable in diverse scientific domains
- The overall evaluation model should also support data-flow processing (i.e. pipeline evaluation), in addition to query processing.
- The main aspects of global optimization should deal with interleaved pipelined evaluation of series of inter-related products, and concentrate on optimizing throughput via data flow control, buffer management, and materialization supporting clustering and indexing.

Future Action Paths for EOSCUBE:

We elaborate on recommended activities in Section VI. Below is a summary of main paths of action that will have to be carefully discussed and planned with EOSDIS.

Research Path

- Optimization algorithms for workflow (for data flow evaluation)
- Optimization for quasi-views, which dynamically control with each product query, when the evaluation is postponed and when is restarted
- Specialized techniques for spatio-temporal indexing and clustering
- Optimization of materialized views, which support (especially created) intermediate results to support clustering and indexing

- GIS constraint algebras, which will support interoperability with GIS under unified constraint model.

Industrial-strength implementation path

- CCUBE/EOSCUBE core, focusing on performance for individual queries.
- Pipeline evaluation model.
- ODBC support, and through it, a range of object managers and DBMS.
- Platforms support, including mass storage system.
- Workflow optimization module.
- GIS integration.

Collaborative work with Earth scientists on a specific set of new products, and continued customization of EOSCUBE for them. This will also used as a leverage for later massive deployment of EOSCUBE.

Deployment of EOSCUBE to Centers and Support

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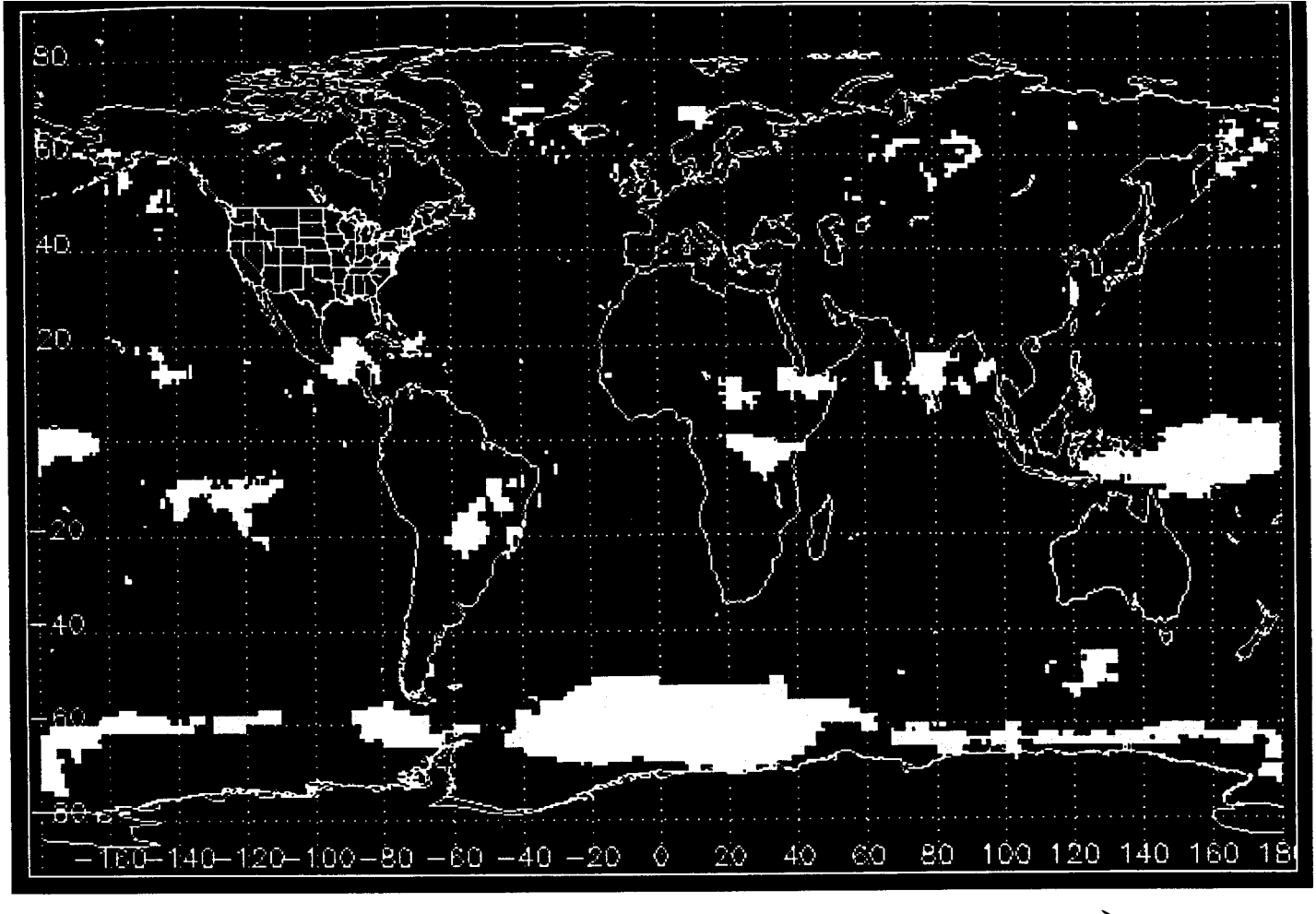
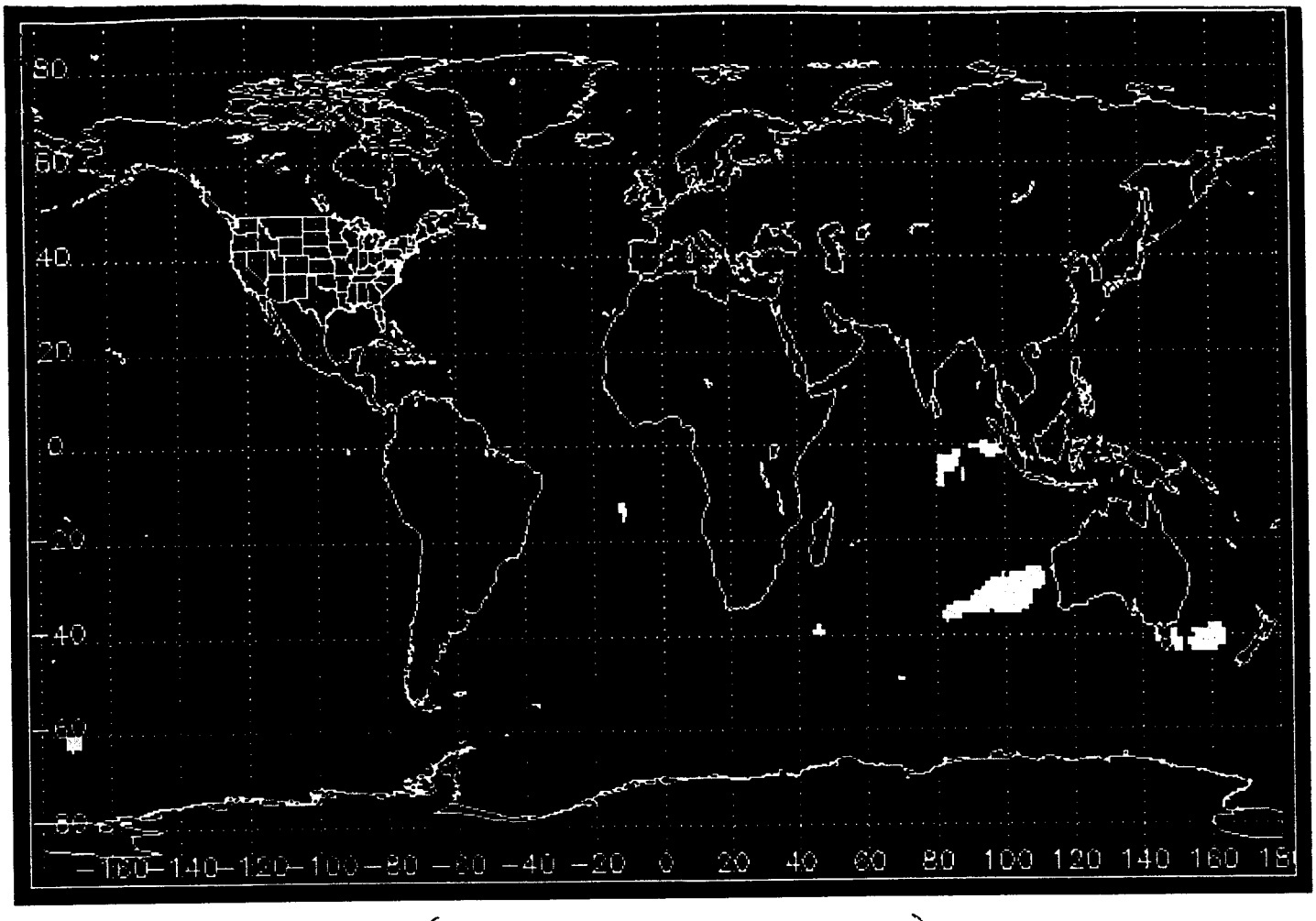
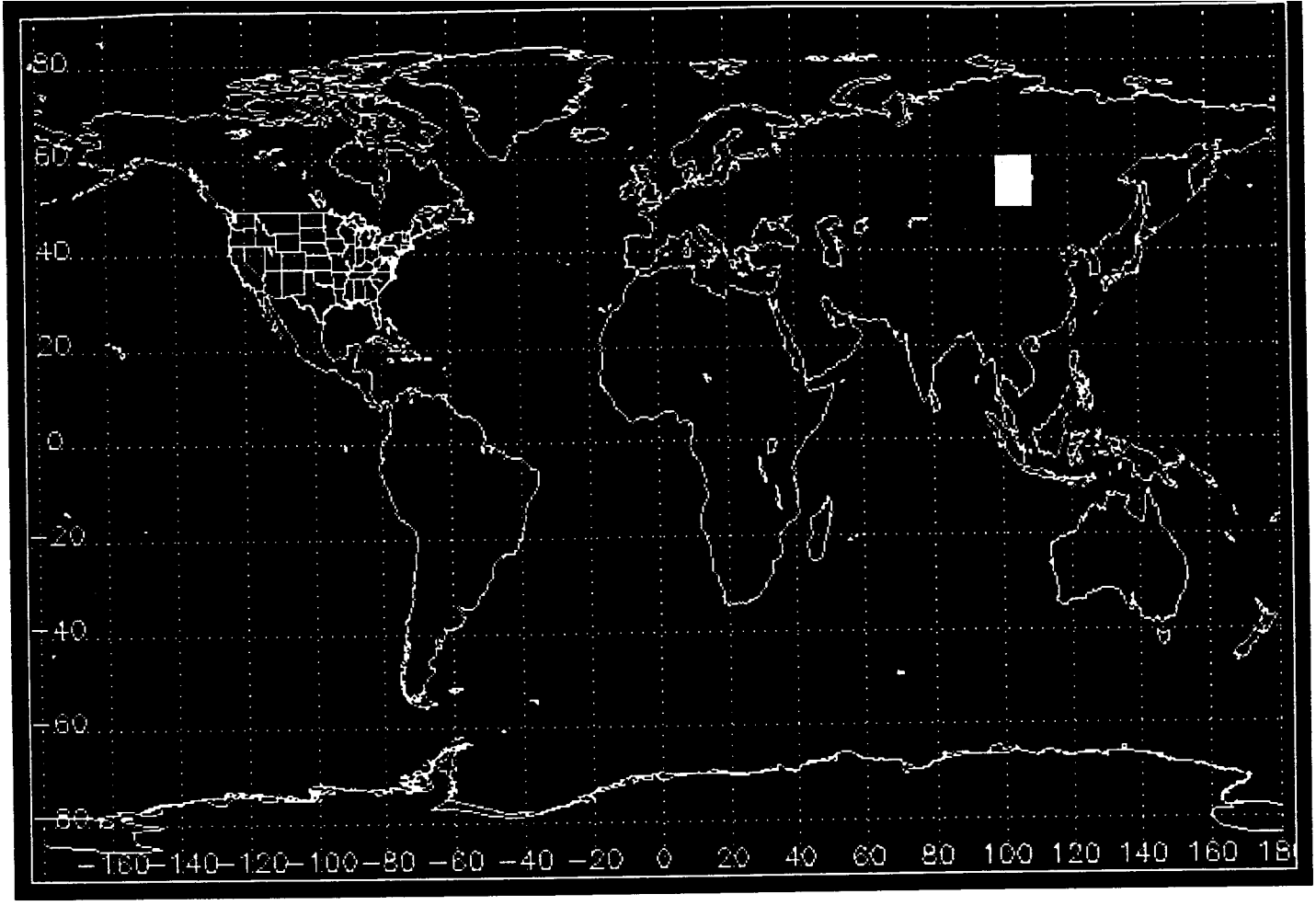
Appendix A



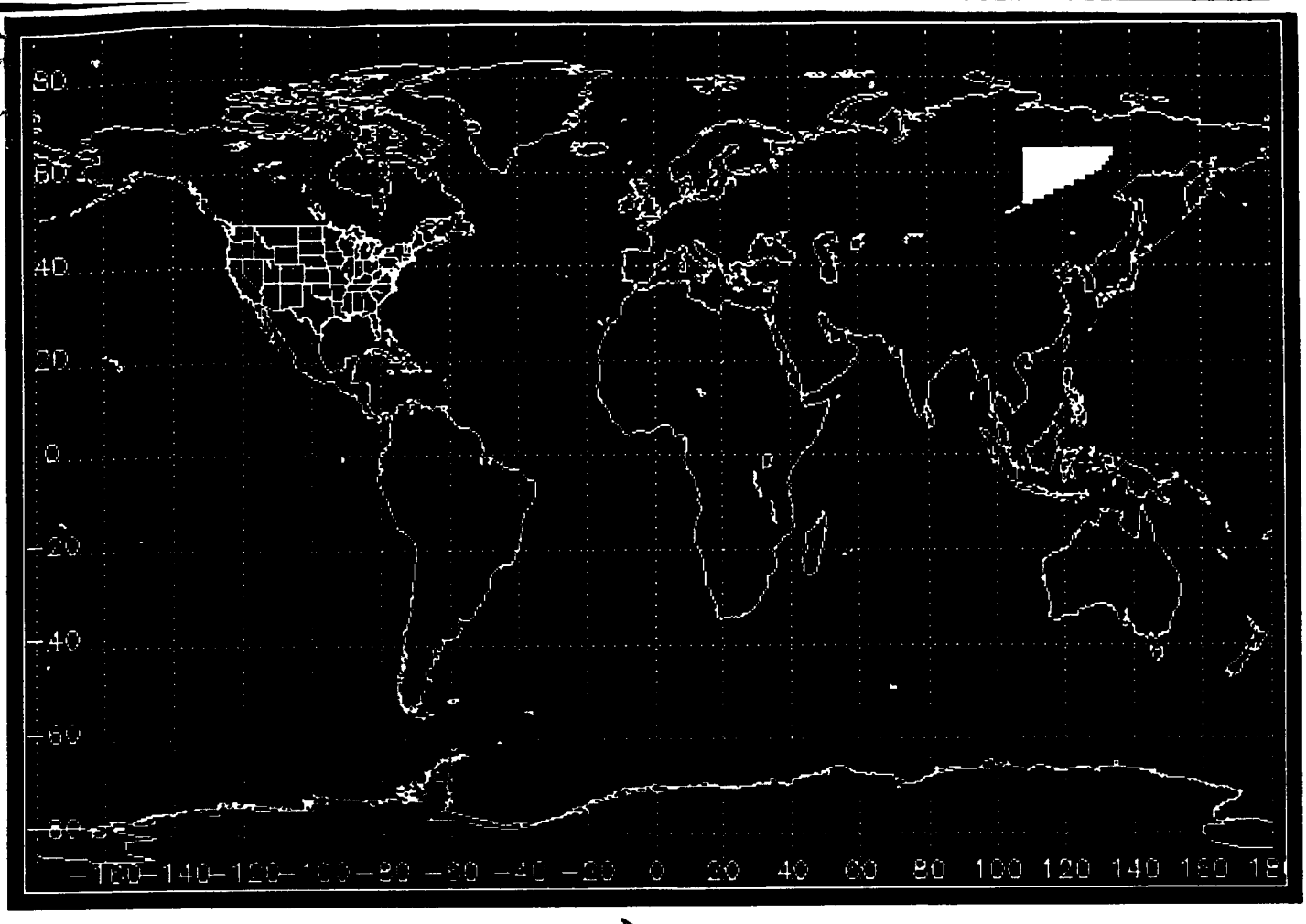
Query 5 (SST, CLD correl. ≥ 0.6)



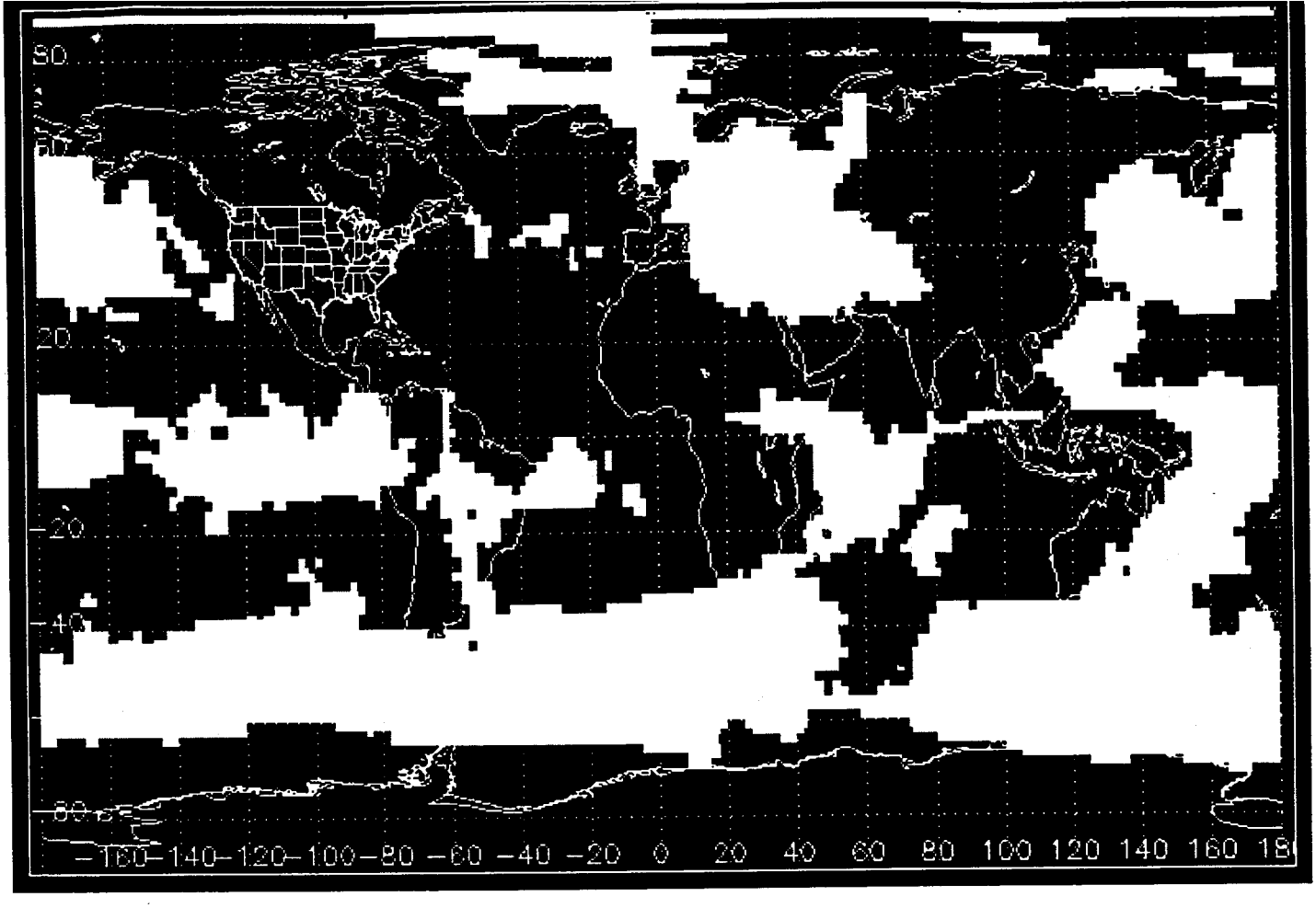
Query 3 (SST difference $\geq$ 2.0)



Query 4 (input area)



Query 1 (input area)



Query 2 (cld ≥ 75.0)

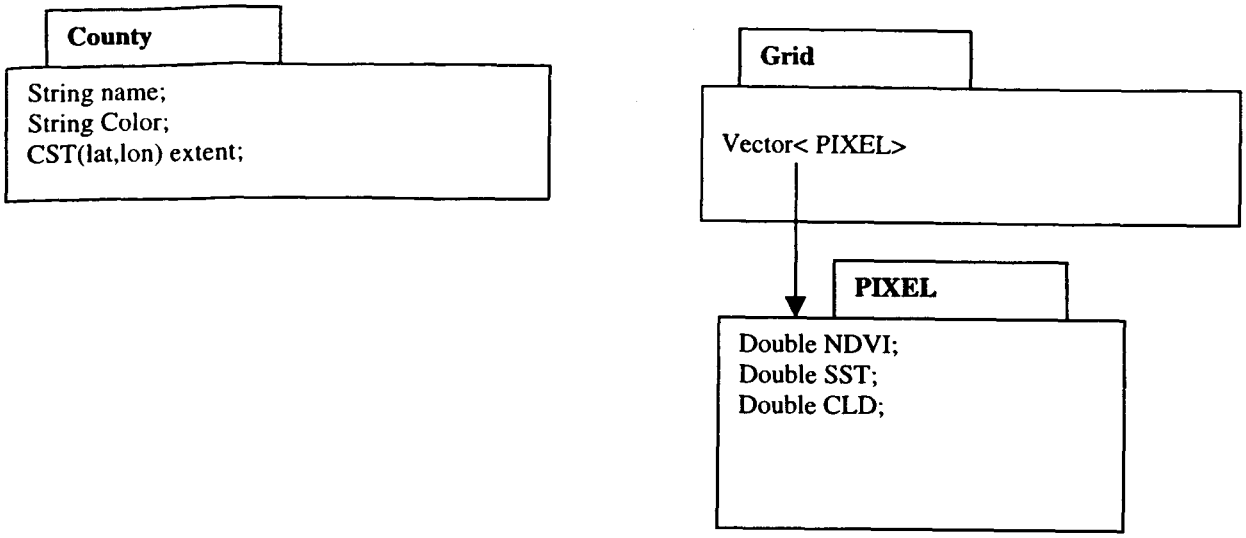
Query 1 output:

0.0846923

Query 4 output:

0.35008 0.235478 0.0327797 0.0294338 0.056053 0.0490407 0.0870465 0.0434318 0
.000460318 0.2558 0.321847 0.156925 0.0482001 0.159197 0.130963 0.149418 0.10525
7 0.021373 0.0475394 -0.0249535 -0.012261 0.0866486 0.317112 -0.0618164

Appendix B



Simplified Schema

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Task 87

16. Abstract

The EOSCUBE constraint database system is designed to be a software productivity tool for high-level specifiction and efficient generation of EOSDIS and other scientific products. These products are typically derived from large volumes of multidimensional data which are collected via a range of scientific instruments.

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